Adaptive Control Strategies for Process Control: A Survey

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In recent years there has been extensive interest in feedback control systems that automatically adjust their controller settings to compensate for changes in the process or the enviornment. Such systems are referred to as adaptive controllers. This survey paper reviews the current state of the art in adaptive control from a process control perspective and describes leading design techniques. Potential operating problems associated with adaptive control schemes are considered. A survey of experimental applications of adaptive control systems to process control problems is also included.

SCOPE

Process control systems inevitably include adjustable controller settings that facilitate process operation over a wide range of conditions. Typically, controller settings are tuned after the control system has been installed using time-consuming, trial-and-error procedures. If process conditions change significantly, then the controller must be retuned in order to obtain satisfactory control.

In recent years, there has been extensive interest in adaptive control systems that automatically adjust the controller settings to compensate for unanticipated changes in the process or the environment. Adaptive control schemes provide systematic, flexible approaches for dealing with uncertainties, nonlinearities, and time-varying process parameters. Consequently, adaptive control systems offer significant potential benefits for difficult process control problems where the process is poorly understood and/or changes in unpredictable ways. The practical benefits of adaptive control have been documented in a wide variety of successful industrial applications.

Although adaptive control has been a reputable research area for about thirty years, it is only in the last decade that it has achieved prominence as one of the most active areas in the field of control engineering. Current, extensive efforts in adaptive control are due to two developments:

1. Significant progress in control theory and the development of practical adaptive control algorithms.
2. Breakthroughs in microelectronics that have made it possible to implement adaptive control schemes in a simple and inexpensive manner.

This survey article on adaptive control is intended both as a tutorial for the nonspecialist and as a critical evaluation of existing design techniques. Emphasis is placed on fundamental concepts and alternative design strategies rather than detailed derivations and mathematical rigor. Three popular design strategies for adaptive controllers are considered in detail: self-tuning controllers, stability-based methods (e.g., model reference adaptive control), and pole placement techniques. Potential operating problems associated with current adaptive control schemes are also considered. Since many adaptive control schemes are based on identifying a process model on-line, a critical review of recursive parameter estimation techniques is also included.

CONCLUSIONS AND SIGNIFICANCE

During the past decade, impressive progress has been made in the theory and application of adaptive control. Design techniques based on cost function minimization, pole assignment, and stability theory have reached a mature state of development. A great deal of practical experience has been gained with adaptive control systems in both simulation and experimental studies. As indicated in Tables 2 to 4, a significant number of industrial applications have already been reported and commercial adaptive controllers are now available.

Adaptive control strategies provide a promising approach for poorly understood processes and for processes with nonlinear behavior and time-varying dynamics. However, in order for adaptive control systems to have a major impact on industrial practice, two key problems need to be resolved:

1. The adaptive controllers must be robust enough to perform well over a wide range of conditions.
2. They should be easy for nonexperts to use. In particular, the user should only have to specify a minimal amount of information concerning numerical values of design parameters, desired closed-loop response characteristics, and similar requirements.

If these two problems can be resolved, adaptive control will have a very promising future.

Introduction

The early development of adaptive control strategies was motivated by the design of autopilots for high-performance aircraft and rockets. Since the response characteristics of these aircraft varied significantly during flight conditions, classical linear controllers with constant controller parameters did not always provide satisfactory control. Consequently, considerable research activity during the 1950's was concerned with developing self-adaptive systems that would automatically adjust to changing flight conditions (Gregory, 1959). These early efforts were largely unsuccessful and have been characterized as having "... a lot of enthusiasm, bad hardware, and nonexisting theory" (Åström, 1983). During the 1960's, design strategies for adaptive control were placed on a more secure theoretical basis by the introduction of modern control concepts, especially those from stability theory.

Renewed interest in adaptive control occurred in the 1970's due to significant theoretical developments such as self-tuning control systems (Åström and Wittenmark, 1973; Clarke and Gawthrop, 1975) and the widespread availability of inexpensive digital control hardware (e.g., microprocessor-based systems.) In recent years, general purpose, adaptive control systems have appeared as commercial products in Europe, Japan, and North America (Hoopes et al., 1983; Kraus and Myron, 1984; Bengtsson and Egardt, 1984). Special-purpose systems for specific applications such as cement kilns are also available (Lohja Corp., 1982).

Before launching into a more detailed discussion of adaptive control techniques, it would seem advisable to clarify what is meant by adaptive control. It is somewhat surprising that despite the thousands of papers that have been published on adaptive control, there is no general consensus on a formal definition for the term. Early controversies concerning alternative definitions have been described by Jacobs (1981). For our purposes, we will forego a formal definition and simply regard an adaptive control system as one that automatically adjusts the controller settings to accommodate changes in the process to be controlled or its environment.

It is convenient to distinguish between two general categories of adaptive control problems. The first category consists of problems where the process changes cannot be directly measured or anticipated. Most of the adaptive control literature has emphasized this type of problem. The second category consists of control problems where process changes can be anticipated or inferred from process measurements. In these situations, if the process is reasonably well understood, it is feasible to adjust the controller settings in a predetermined manner as process conditions change. For example, a "table look-up" approach could be adopted where different sets of controller constants are stored for a variety of different operating conditions. Thus, if a grade change were made or if the process throughput were changed, a new set of controller constants would be used. A simple strategy for control problems where the process gain K_p varies in a known or measurable manner is to maintain the product $K_c K_p$ constant, where K_c is the controller gain. In principle, this approach will maintain an adequate margin of stability despite variations in the process gain.

These two examples have illustrated a simple type of adaptive control strategy referred to as gain scheduling. The term is used because this approach was initially used to accommodate changes in the process gain only (Åström, 1983). Gain scheduling has been very effective in a variety of industrial applications, especially pH control (Shinskey, 1979). In recent years, digital controllers with gain scheduling options have become commercially available (Andreiev, 1981). However, this approach is limited by the need to relate process changes to variables that can be measured on-line. Furthermore, the simple gain scheduling approach of keeping $K_p K_c$ constant may result in poor control unless the process dynamics are also considered. For example, if the process contains a long time-delay, standard gain scheduling may be worse than conventional PID (proportional-integral-derivative) control unless some type of time delay compensation is employed (Wong and Seborg, 1985a).

The remainder of this paper is concerned with adaptive control strategies for the more difficult class of problems where process changes are unpredictable and cannot be directly inferred from process measurements. The wide variety of adaptive control techniques that have been developed for this class of problems can be conveniently (but somewhat arbitrarily) divided into four categories:

1. Adaptive controllers designed using a quadratic cost function (e.g., self-tuning regulators and controllers).
2. Design methods based on stability theory (e.g., model reference adaptive control).
3. Pole-zero assignment techniques (e.g., Vogel-Edgar approach).
4. Miscellaneous approaches.

These techniques will be described in detail and critically evaluated later in separate sections of the paper. However, it should be emphasized here that the delineations between the four categories are not as clear-cut as may first appear. For example, recent research has demonstrated that although self-tuning controllers and model reference adaptive control have been developed from different design viewpoints, the techniques are closely interrelated (Egardt, 1980; Shah and Fisher, 1980; Landau, 1982; Ljung and Landau, 1978). They may also be analyzed from a unified theoretical framework (Egardt, 1980; Goodwin and Sin, 1984).

Self-tuning control

A general strategy for designing adaptive control systems is to estimate model parameters on-line and then adjust the controller settings based on the current parameter estimates. This approach is often referred to as self-tuning control and was first proposed by Kalman (1958). A block diagram of a self-tuning control system is shown in Figure 1. At each sampling instant the parameters in an assumed dynamic model are estimated recursively from input-output data and the controller settings are then updated. The self-tuning approach has received more attention than any other adaptive control strategy during the past decade. In particular, it provides the basis for the self-tuning regulator (Åström and Wittenmark, 1973), the self-tuning controller (Clarke and Gawthrop, 1975), and the pole placement techniques to be discussed later in this paper.

The self-tuning control configuration in Figure 1 is flexible enough to accommodate a wide variety of parameter estimation techniques and controller design strategies. Typically, the dynamic model is assumed to be a linear difference equation model with constant parameters. Recursive least squares and extended least squares approaches have been the most widely used parameter estimation techniques, but recursive versions of other methods such as maximum likelihood and instrumental variables have also received attention (Åström et al., 1977; Isermann, 1982).

In self-tuning control systems the controller is typically designed either to minimize a quadratic cost function or to place the poles (and perhaps zeros) of the closed-loop system at desired locations. But other control design methods such as deadbeat control can also be utilized (Isermann, 1981; 1982). In general, self-tuning control systems do not have the classical PID structure; however, several self-tuning PID controllers have recently been proposed (Wittenmark, 1979; Wittenmark and Åström, 1980; Bányász and Keviczky, 1982; Gawthrop, 1982a; Cameron and Seborg, 1983) and at least one is commercially available (Hoopes et al., 1983; Hawk, 1983).

Classification of design methods

It is convenient to classify self-tuning control techniques into two general classes: explicit or indirect methods, and implicit or direct methods. In the explicit approach a process model is employed and the control calculations are based on the estimated model parameters. This is also referred to as an indirect approach because the model parameters do not directly appear in the control law. In the implicit or direct approach, the original process model is converted to a predictive form that allows the future process output to be predicted from current and past values of the input and output variables. By using a predictive model, the control calculations are eliminated since the model parameters are also used as the control law parameters. This approach is referred to as a direct method because the control law parameters are directly updated from input-output data. It is also referred to as an implicit method because the process model is implicitly included in the control law. The above classification scheme is convenient but somewhat arbitrary, since it has been demonstrated (Narendra and Valavani, 1979) that for a particular parameterization of the process model, both direct and indirect methods result in identical equations.

Table 1 provides an overview of the literature on adaptive

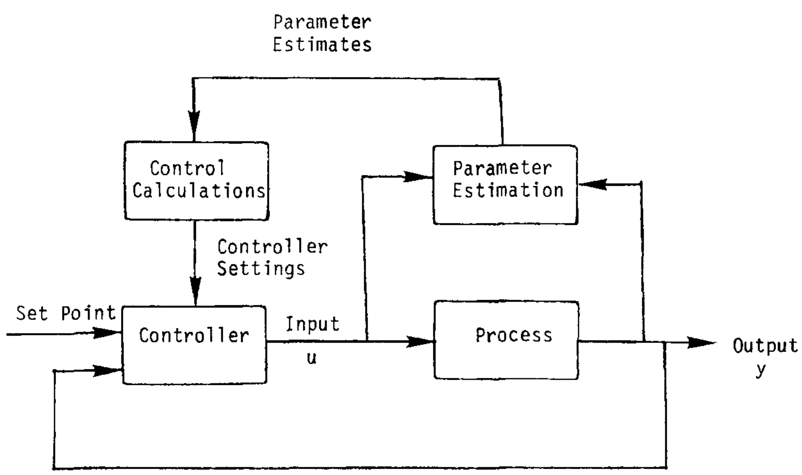

Figure 1. Self-tuning control system.

Table 1. An Overview of Recent Adaptive Control Literature

Tutorial Articles	General References
Åström (1980a,b; 1983)	Anderson and Ljung (1984)
Bélanger (1982)	Åström (1982)
Clarke (1981a,b; 1984)	Goodwin and Sin (1984)
Isermann (1982)	Harris and Billings (1981)
Jacobs (1981)	Landau (1979)
Wellstead and Zanker (1982)	Unbehauen (1980)
Wittenmark and Åström (1984)	

Survey Articles

Bibliography on adaptive control: Asher et al. (1976)
Applications of adaptive control: Parks et al. (1980)
Self-tuning control systems: Åström et al. (1977); Wittenmark (1975); Åström (1983)
Model reference adaptive control: Landau (1974); Narendra and Peterson (1980)

Conference Proceedings

IEEE Conf. on Applications of Adaptive and Multivariable Control, Hull, England (1982)
IFAC Workshop on Adaptive Systems in Control and Signal Processing, San Francisco (1983)
Yale University Workshops on Applications of Adaptive Systems Theory (1979, 1981, 1983, 1985)
IFAC Workshop on Adaptive Control of Chemical Processes, Frankfurt, West Germany (1985)

control. Excellent tutorial articles are available (Jacobs, 1981; Clarke, 1981a, b, 1984; Åström, 1980a, b, 1983; Bélanger, 1982; Wellstead and Zanker, 1982; and Isermann, 1982). Three publications are especially recommended because they provide an overview of current design methods and representative applications: Harris and Billings (1981), Åström (1982), and Anderson and Ljung (1984). Landau (1979) describes developments in model reference adaptive control through 1977. Goodwin and Sin (1984) provide a unified treatment of the adaptive control field. Applications of adaptive control techniques have been surveyed by Parks et al. (1980). Current research efforts are described in the proceedings of several recent conferences on adaptive control, shown in Table 1.

On-Line Parameter Estimation

Adaptive control is usually based on simultaneous model identification and control. Adaptive control is normally employed when the actual process is nonlinear and/or high-order; approximate models, usually linear and low-order, must be employed for the purposes of implementation. Use of an approximate model is a practical approach because appropriate controllers for nonlinear, stochastic processes are not easily calculated in real time.

This section covers the topics of model selection and on-line parameter estimation for the purposes of adaptive control. First we describe the types of models used (linear difference equations) and the parameterization of these models. Next, algorithms for recursive least squares estimation of model parameters are presented, along with suggested modifications that can improve the performance of on-line estimators. Operational experience with such items as covariance resetting, forgetting factors, use of a perturbation signal, and estimator diagnostics is reviewed. Finally, we discuss the extension of recursive estimation to multiple input–multiple output processes.

Linear difference equation models

Linearization of the process model is a generally accepted procedure in control theory and thus has been the basis of most adaptive control algorithms. A typical single input–single output (SISO) model employed in adaptive control is a linear difference equation, the so-called ARMAX model (autoregressive, moving average model with auxiliary or exogenous input; see Goodwin and Sin, 1984; Ljung and Söderström, 1983).

$$\begin{aligned} y(t) + a_1 y(t-1) + \cdots + a_n y(t-n) &= b_0 u(t-k) \\ &+ b_1 u(t-k-1) + \cdots + b_m u(t-k-m) \\ &+ c_0 \xi(t) + c_1 \xi(t-1) + \cdots + c_n \xi(t-n) + d(t) \end{aligned} \quad (1)$$

where y is the output, u is the input, ξ is a stochastic noise variable (random variable with normal distribution and zero mean), d is the load disturbance variable (usually unmeasured), and t is a nonnegative integer which denotes the sampling instant, $t = 0, 1, 2 \ldots$. In Eq. 1 n and m are known positive integers, and k is the known time delay expressed as an integer multiple of the sampling period ($k \geq 1$). The model parameters, a_i, b_i, and c_i may be unknown. By introducing operator notation, Eq. 1 can be written more compactly as

$$A(q^{-1})\, y(t) = B(q^{-1}) u(t-k) + C(q^{-1}) \xi(t) + d(t) \quad (2)$$

where q^{-1} is the backward shift operator, $q^{-1} y(t) = y(t-1)$, and the A, B, and C polynomials are defined by

$$A(q^{-1}) = 1 + \sum_{i=1}^{n} a_i q^{-i}$$

$$B(q^{-1}) = \sum_{i=0}^{m} b_i q^{-i}$$

$$C(q^{-1}) = \sum_{i=0}^{n} c_i q^{-i}$$

Equations 1 and 2 are linear difference equations which are referred to as stochastic, discrete-time models. In fact, these equations provide canonical representations for sampled-data systems with white noise disturbances (Åström, 1970). While the form of the model is linear, in adaptive control the model coefficients are assumed to be time-varying and are estimated in real time. Hence, Eq. 1 does not amount to the traditional Taylor series linearization of the true process model (Jacobs, 1981). In fact, the coefficients are uncertain, which introduces a nonlinear influence into the model. A few studies have been reported using particular classes of nonlinear models such as bilinear models (Svoronos et al., 1981; Balestrino et al., 1984) or Hammerstein models (Anbumani et al., 1981; Lachmann, 1982).

Linear, discrete-time models are preferred for adaptive control because they lead to algorithms that are readily implemented on a digital computer. As is well known, there is a loss of information when a continuous process is subjected to the sampling operation. However, this is not a practical difficulty as long as the sampling time is about one-tenth of the dominant time constant (Clarke, 1981b) or $\frac{1}{15} t_{95} \leq \Delta t \leq \frac{1}{4} t_{95}$, where t_{95} is the 95% settling time (Isermann, 1982). The actual choice may also be influenced by the process time delay, as discussed below.

Four model parameters in Eq. 1 must be specified *a priori:* model orders n and m, time delay k, and sampling period Δt. The selection of difference equation order is somewhat arbitrary, with n typically chosen to be 2 or 3. Too large a value of n is undesirable because this requires more computational effort for on-line parameter estimation. Too small a value of n may not adequately describe the process dynamics, and underestimating the model order may cause the parameter estimates to become unrealistic (Rohrs et al., 1982). The time delay parameter k depends on the ratio of the time delay θ to the sampling period Δt. In general k should be selected so that $k \geq (\theta/\Delta t) + 1$ ($\theta = 0$ corresponds to $k = 1$, i.e., the inherent unit delay in discrete system representation).

Clearly, the choices of k and Δt interact. The sampling period is often chosen so that k has a value of 2 or 3; this tends to reduce the computational effort of various control algorithms (see the later section on quadratic cost function design methods). Choosing Δt to be very small has two disadvantages. First, small values of Δt can cause the process model to become nonminimum phase (Åström et al., 1984; Clarke, 1981a, b). In fact, a fractional time delay leads to a $B(q^{-1})$ polynomial with a zero outside the unit circle (Gawthrop, 1980a, b; Åström et al., 1984; Åström, 1983). Second, if Δt is very small then the control action tends to become excessive (Wellstead and Zanker, 1982). On the other hand, if Δt is too large the controller may respond too slowly to load and set-point changes. However, Rohrs et al. (1985) have demonstrated that slow sampling improves the robustness of the adaptive controller. It may be advantageous in some cases to adjust Δt on-line (McDermott and Mellichamp, 1983).

If the time delay is unknown or varies significantly, either the B polynomial can be expanded, or k can be estimated on-line (Kurz and Goedecke, 1981). If the time delay varies between known limits, $k_{min} < k < k_{max}$, the order of the B polynomial should be increased from m to $m + k_{max} - k_{min}$. Any leading coefficients in $B(q^{-1})$ that are estimated to be zero or near-zero may signify a change in the time delay. This strategy has been successfully employed for pole-shifting techniques (Vogel and Edgar, 1982a, b; Wellstead and Zanker, 1982). Chien et al. (1983) have proposed a related strategy for a modified version of the self-tuning controller (STC). However, simulation studies by Lee and Hang (1985) have shown that this approach can be sensitive to noise and unmeasured disturbances. It may be necessary to employ special methods such as those discussed in the later section on modification of recursive least squares estimation in order to obtain satisfactory results for a varying time delay.

Recursive least squares estimation

The goal in process identification is to infer a model (and estimates of the model parameters) given a data record. This activity can be carried out in an off-line manner, in which all data are analyzed at once, or by using on-line techniques, where the addition of a new data point (or data set) is employed to update the model parameters. In adaptive control, real-time (or sequential) updating of the model parameters is more appropriate than batchwise (nonsequential) processing of the input-output data. Algorithms that are suited to real-time usage and are based on successive updating of the model parameters are called recursive. There are a large number of recursive identification algorithms described in the literature (Ljung and Söderström, 1983); the most popular technique is recursive least squares (RLS).

As a simplification, suppose the noise model parameters, c_i, are set to zero. In this case Eq. 1 can be written in the following vector form:

$$y(t) = \psi^T(t-1)\,\theta(t-1) + \epsilon(t) \tag{3}$$

where (data) the information or regressor vector ψ and parameter vector θ are defined as

$$\psi^T(t-1) = [y(t-1), y(t-2), \ldots u(t-n), \\ u(t-k-1), \ldots u(t-k-m-1), 1]$$

$$\theta^T(t-1) = [a_1, a_2, \ldots a_n, b_1, b_2, \ldots b_m, d].$$

$\epsilon(t)$ represents an error that is assumed to be statistically independent of the inputs and outputs.

The parameter estimation problem is to find the estimates ($\hat{\theta}$) of the unknown parameters (θ) which minimize the loss function

$$J = \sum_{t=1}^{n_d} (y(t) - \hat{y}(t))^2 \tag{4}$$

where $\hat{y}$ is the predicted value of the output based on $\hat{\theta}$, y is the actual value, and n_d is the number of data points. The estimation error is $y - \hat{y}$. In the least squares method, we choose to minimize what is unexplained by the model (the prediction error). The least squares solution ($\hat{\theta}$) can be obtained by collection and analysis of all data taken (Clarke, 1981a).

The equations for recursive least squares computation of the unknown parameters are as follows (Clarke, 1981a; Ljung and Söderström, 1983):

$$\hat{\theta}(t) = \hat{\theta}(t-1) \\ + P(t)\,\psi(t-1)[y(t) - \psi^T(t-1)\,\hat{\theta}(t-1)] \tag{5}$$

where P is called the covariance matrix of the estimation error [dimension $(n + m + 1) \times (n + m + 1)$]. $P(t)$ is a positive definite measure of the estimation error and its elements tend to decrease as t increases; it is calculated using the recurrence relationship:

$$P(t) = P(t-1) - P(t-1)\,\psi(t-1)\,[\psi^T(t-1) \\ \cdot P(t-1)\,\psi(t-1) + 1]^{-1}\,\psi^T(t-1)\,P^T(t-1) \tag{6}$$

Equation 6 requires an initial guess for $P(0)$. Using a matrix inversion lemma, Eq. 6 becomes

$$K(t) = \frac{P(t-1)\psi(t)}{1 + \psi^T(t)P(t-1)\psi(t)} \tag{7}$$

$$P(t) = [I - K(t)\psi^T(t)]P(t-1) \tag{8}$$

I is the identity matrix. Equation 5 can be written as

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\,[y(t) - \hat{y}(t)] \tag{9}$$

where $K(t)$ is a Kalman (feedback) filter gain; this gain multiplies the prediction error to yield the correction term for the model parameter vector. If the noise $\epsilon(t)$ has zero mean, the parameter estimates ($\hat{\theta}$) are unbiased, i.e., $E[\hat{\theta}] = \theta$, the true parameter vector. For white noise ($c_0 = 1$; all other c_i in Eq. 1 are zero), $\hat{\theta}$ is also the minimum variance estimate.

If the system noise $\epsilon(t)$ is not independent but described by a linear difference equation such as Eq. 1, i.e., $c_i \neq 0$, the estimated values of θ are no longer unbiased, since $\psi(t)$ in Eq. 3 now is correlated with $y(t)$. While biased estimates may not be a practical problem for high signal-to-noise ratios, significant difficulties may arise for ratios of 10:1 or less [signal-to-noise ratio is defined as the ratio of variance of the signal (σ_s^2) to the variance of the random input disturbance (σ_N^2)]. One means of handling an estimate which contains bias is the instrumental variable (IV) method (Young, 1970), in which a linear transformation is used to obtain variables that have uncorrelated residuals. If the noise model in Eq. 1 is to be estimated, an alteration in the recursive least squares procedure must be implemented. Extended least squares (Clarke, 1981a) utilizes the fact that if all $\xi(t)$ were known, recursive least squares could be applied. Thus this approach is sometimes referred to as pseudolinear regression, or PLR (Ljung and Söderström, 1983; Goodwin and Sin, 1984). The values of $\xi(t)$ are approximated by estimates of $\epsilon(t)$ (Eq. 3), using current model parameter estimates, $\hat{\theta}(t)$. This allows the use of an extended parameter vector of a_i, b_i, and c_i. Convergence cannot be proved for all types of C polynomials, but this is not a serious practical shortcoming, based on applications experience. A method that does converge is recursive maximum likelihood (Åström et al., 1977; Isermann, 1982), in which the estimator is based on a nonlinear programming algorithm. A simpler method which also converges is AML, or approximate maximum likelihood (Solo, 1979). For a summary of key convergence results in recursive estimation, see Ljung and Söderström (1983) and Goodwin et al. (1984).

In order to simulate Eqs. 7, 8, and 9, a suitable choice of $P(0)$ [and $\hat{\theta}(0)$] must be made. For $P(0)$ a diagonal matrix with large elements (e.g., 10^4 or larger) implies that the user's confidence in $\hat{\theta}(0)$ is poor, while small values for the diagonal elements imply that $\hat{\theta}(0)$ is a good estimate. Large values of the diagonal elements of $P(0)$ will cause rapid changes in $\hat{\theta}(t)$ initially, while small values cause $\hat{\theta}(t)$ to change more slowly (Young, 1969). Another characteristic of Eqs. 7 and 8 is that the norms of P and K ($\|P\|$, $\|K\|$) tend to zero as more data are processed, meaning that corrections to $\hat{\theta}$ become smaller. This leads to convergence of the parameters, a desirable result if the parameters are indeed constant. However, most chemical processes tend to have model parameters that are at least slowly time-varying. An adaptive controller/estimator should have the capability to track parameter changes even if the estimator has become ill-suited to monitor such changes. This problem of loss of sensitivity (sometimes referred to as "falling asleep") is more severe for closed-loop parameter estimation (estimation under feedback control) than for open-loop estimation; we discuss this subject more fully in this section.

Modification of recursive least squares estimation

The least squares parameter estimation algorithm can be modified to maintain its sensitivity to process parameter variations. Perhaps the most common modification is to weight new data more heavily than old data. This can be done by including an exponential weighting factor (called a forgetting factor) in the performance index:

$$J(\theta(t)) = \sum_{i=1}^{t} \lambda^{t-i}\,[\psi^T(i-1)\,\theta(i) - y(i)]^2 \tag{10}$$

where λ is the exponential weighting factor, $0 < \lambda \leq 1$. When $\lambda = 1$ all data are weighted equally (see Eq. 4). For $0 < \lambda < 1$, more weight is placed on recent measurements than on older measurements. Following the derivation shown by Young (1969) for the previous algorithm, the performance index given by Eq. 10 results in the following recursive least squares algorithm:

$$P(t) = \frac{1}{\lambda}\{P(t-1) - P(t-1)\,\psi(t-1)[\psi^T(t-1) \cdot P(t-1)\,\psi(t-1) + \lambda]^{-1}\,\psi^T(t-1)\,P(t-1)\} \tag{11}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\psi(t-1)[y(t) - \psi^T(t-1)\hat{\theta}(t-1)] \tag{12}$$

It can be seen from Eq. 11 that the effect of the exponential weighting factor, λ, is to prevent the elements of P from becoming too small. This maintains the sensitivity of the algorithm and allows new data to continue to affect the parameter estimates. On the other hand, when x and u are close to zero, then $P(t-1)\psi(t-1) \rightarrow 0$, and $P(t) = P(t-1)/\lambda$. Hence P grows exponentially until ψ changes. Equation 12 shows how bursts in $\hat{\theta}(t)$ can occur for large P, especially when the set point is changed or a perturbation signal is introduced. This phenomenon is known as estimator windup or covariance windup (Åström, 1982).

A second modification to improve sensitivity of the parameter estimates, suggested by Young (1969), is to add a positive definite matrix, D, to $P(t-1)$, creating an *a priori* covariance matrix, $P(t/t-1)$, to be used in place of $P(t-1)$ in Eq. 13. With this modification, the algorithm given in Eqs. 13 and 14 becomes:

$$P(t/t-1) = P(t-1) + D \tag{13}$$

$$P(t) = \frac{1}{\lambda}\{P(t/t-1) - P(t/t-1)\psi(t-1) \cdot [\psi^T(t-1)P(t/t-1)\psi(t-1) + \lambda]^{-1}\,\psi^T(t-1)P(t/t-1)\} \tag{14}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t/t-1) \cdot \psi(t-1)[y(t) - \psi^T(t-1)\hat{\theta}(t-1)] \tag{15}$$

Equation 15 indicates that addition of D can prevent $\|P\|$ from becoming too small. D can be chosen as a diagonal matrix with the magnitude of the elements depending upon the expected rate of variation of the parameters. An analogous technique is covariance resetting (Goodwin et al., 1983; Dumont and Bélanger, 1981), where the covariance matrix is reinitialized as necessary (i.e., when $\|P\|$ becomes small). In subsequent discussion of the use of D, the reader should recognize that these two techniques are equivalent.

The use of the least squares estimation algorithm is sometimes limited by the numerical accuracy of the particular computer used. This may be a significant problem for microprocessor implementation of recursive least squares estimation. Let us illustrate the problem for a model with a single parameter θ. Defining $\tilde{\theta}(t)$ to be the parameter error, $\theta - \hat{\theta}(t)$, Eq. 9 can be written for this one-dimensional case,

$$\tilde{\theta}(t) = [1 - K(t)\psi(t)]\tilde{\theta}(t-1) + K(t)\epsilon(t) \qquad (16)$$

This stochastic difference equation can be come unstable if $|1 - K(t)\psi(t)| > 1$, or when $K(t)\psi(t)$ becomes negative. This is tantamount to P (a scalar for the one-dimensional case) becoming negative; in the multidimensional case, P may become indefinite due to round-off. Once this happens, the recursive calculation becomes unstable. Round-off problems may occur after 5,000 to 10,000 iterations with a 32 bit floating-point representation (Clarke, 1981b). One modification which solves the numerical instability problem is called square root filtering (Bierman, 1977), where the covariance matrix is factored as

$$P(t) = S(t)\,S^T(t) \qquad (17)$$

where S is an upper-triangular matrix called the square root of P. $S(t)$ is then incorporated into the algorithm, and $S(t)$ is adjusted at each iteration. This ensures that P will be positive definite. A second technique is to decompose $P(t)$ into a product of upper-triangular and diagonal matrices. See Peterka (1975), Bierman (1976, 1977), and Morris et al. (1981) for more details on the appropriate equations and computational experience.

Selective updating of certain model parameters is sometimes necessary for models where m or n is large. In the case of discrete convolution (or impulse response) models, as many as 30 parameters may need to be updated. Asbjornsen (1984) has proposed an estimation algorithm for this case that adjusts only those parameters which give a significant improvement in the residual of the model fit. If the improvement is insignificant, the estimates are not changed.

Potential operating problems of parameter estimation algorithms

A certain amount of tuning and operational experience with parameter estimation algorithms is required to make them successful, since certain operational problems may occur during implementation, due to real world conditions. When the controlled process is operating satisfactorily, very little excitation of the process occurs. As discussed earlier, small model errors can lead to large parameter changes (see Eq. 12), causing oscillation in the process variable. Fortunately, such excitation of the system will lead to improved estimation, followed by improved control. Anderson (1985) has shown that bursting phenomena can result from noise or unmodeled dynamics in the absence of persistent excitation, quite apart from the estimator windup effect.

Another particularly difficult problem in estimation occurs when a set-point change is implemented in a nonlinear system; an equivalent situation arises when an unmeasured disturbance suddenly changes. This change in operating point imparts a sudden or jump change to the estimated model parameters, as opposed to the slowly changing parameters normally assumed (parameter drift). This phenomenon has been observed by many investigators, e.g., Vogel and Edgar (1982a) and Fortescue et al. (1981); Figures 2 and 3 (Vogel, 1982) show a typical behavior pattern for an estimator in an experimental application where a load change occurred at $t = 2.6$ min. The large estimation error gives rise to changes in the model parameters (d, b_1, b_2, and b_3 in Figure 3).

There are a number of approaches to deal with such operating problems, including covariance resetting, variable forgetting factor, and use of a perturbation signal. The following three sections discuss these approaches.

Covariance resetting

The performance of a parameter estimation algorithm is a function of the covariance resetting procedure and the use of the forgetting factor, λ. If D is selected with all elements equal to zero and $\lambda = 1$, the algorithm corresponds to the estimation algorithm given in Eqs. 7 and 8, which becomes progressively more insensitive to parameter changes when the process is operating satisfactorily. The sensitivity of the algorithm to parameter changes can be improved by selecting $\lambda < 1.0$ and/or using a diagonal D with positive elements. Although these strategies improve the sensitivity of the algorithm, they have two serious disadvantages. First, if $\lambda < 1$ and/or $D \neq 0$, the algorithm is more sensitive to noise, as well as parameter changes, which causes the parameter estimates to drift erroneously. The quality of the estimates can be improved if a perturbation signal is added to the process input, as discussed later.

The second disadvantage is that with $\lambda < 1$ and/or $D \neq 0$, the elements of P may become excessively large with time. This in turn causes the algorithm to become overly sensitive to parameter changes and noise, resulting in large fluctuations and drifting in the parameter estimates. As mentioned by Åström and Wittenmark (1980), the large values of P may also lead to numerical problems. Through Eq. 16, relatively large fluctuations in the process output cause the elements of P to decrease, but when no disturbances enter the process for some period of time the elements of P may increase. The rate of increase is a function of λ and the elements of D.

The problems discussed above concerning the performance of the estimation algorithm have been observed in both simulation and experimental applications (Vogel and Edgar, 1982a; Clough et al., 1983). It is apparent that simply selecting constant values for λ and D will yield unsatisfactory performance for one reason or another. For the purposes of adaptive control, Vogel (1982) found the best performance for $\lambda = 1$ and $D = 0$.

However, certain conditions may arise when a nonzero D should be added. Vogel (1982) and Goodwin and Teoh (1983) found that the occurrence of a parameter change requiring a change in λ or covariance resetting can be determined by observing the estimation error, $y(t) - \hat{y}(t)$, i.e., λ is kept equal to 1 and the covariance is reset using D in Eq. 15 only if the absolute value of the estimation error exceeds a user-specified limit. This

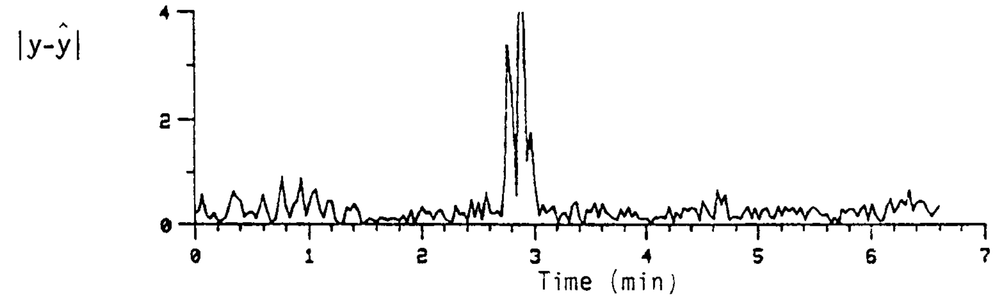

Figure 2. Absolute value of estimation error for adaptive controller applied to the cooling water control loop.

limit should be as small as possible and yet still be larger than the fluctuations due to noise. The addition of a nonzero D increases the elements of P, which allows the parameter estimates to change. Since $D \neq 0$ is added only at a specific time, the elements of the P matrix will again become small with time. There is an optimum resetting interval for drifting parameters (Goodwin and Teoh, 1983); the nonzero D should not be added too frequently since it will cause unnecessary parameter estimate fluctuations and possibly lead to the other problems discussed above. Too-frequent covariance resetting may be avoided by examining the size of the elements of P and adding D only if the elements of P are small. If the elements are not small, addition of D is not necessary anyway. The size of the elements of P may be indicated by the trace of P. Thus, a nonzero D is added only if the trace of P is below a user-specified limit. This means that once D is added, it should not added again until the trace of P drops below the specified limit.

Variable forgetting factor

Another modification that can be employed to improve the sensitivity of an estimator is to adjust the forgetting factor periodically (Fortescue et al., 1981; Ydstie et al., 1985, Ydstie and Sargent, 1982). This strategy is referred to as the variable forgetting factor approach. Clarke (1981b) has suggested that the value of the forgetting factor can be adjusted based on the nature of the expected parameter variations. A value near 1 (e.g., 0.999) implies slow variations, while a smaller value (say $\lambda = 0.95$) implies fast parameter variations. The lower value can be used initially after a parameter change is detected, followed by a gradual increase in λ to 0.999. The weighting factor can be related easily to the observed estimation error, which is analogous to the procedure suggested by Vogel (1982).

Hägglund (1982, 1983a, b) has recently suggested a more sophisticated way of discounting old data, based on a variable forgetting factor, which is adjusted at each iteration so that a constant desired amount of information is retained. This gives an additional parameter to be optimized for any given application environment. However, both Vogel (1982) and Goodwin and Teoh (1983) found that decreasing λ from 1 at specific times did not always yield satisfactory performance.

Fortescue et al. (1981) suggested that λ be kept near unity unless the prediction error becomes large; then λ should be

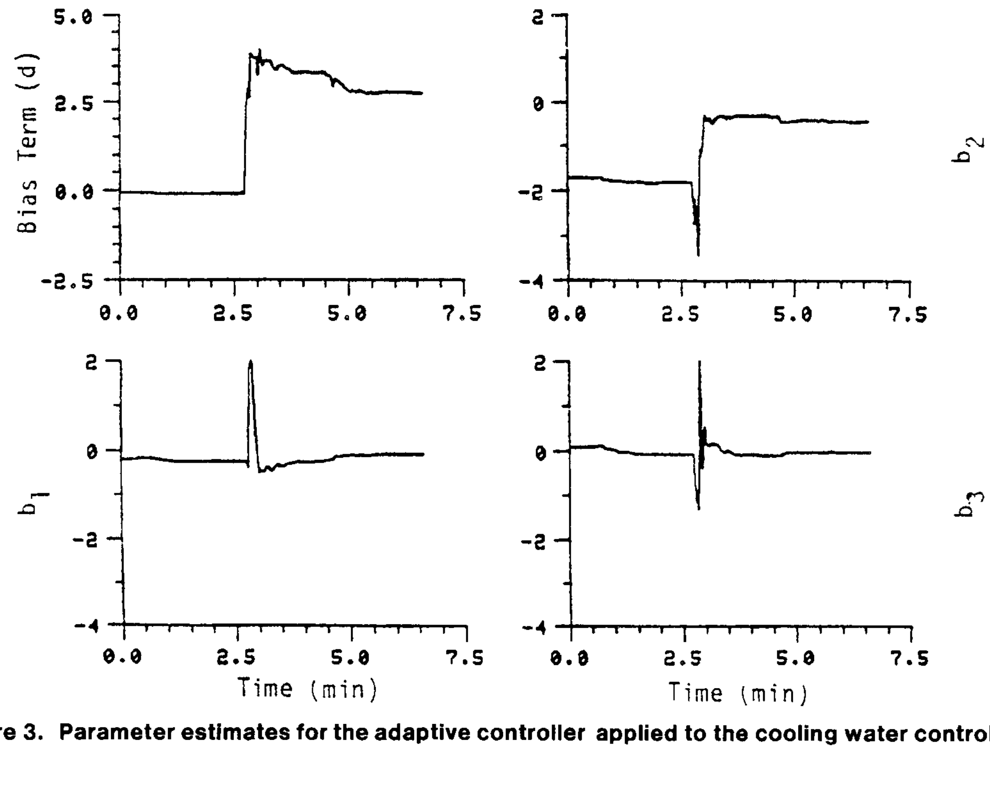

Figure 3. Parameter estimates for the adaptive controller applied to the cooling water control loop.

decreased for several steps thereafter. Ydstie and Sargent (1982), Ydstie et al. (1985), and Ydstie (1982, 1984a, b) have proposed an improved algorithm that keeps the information content of the estimator constant; this algorithm has desirable stability and cnvergence characteristics. The updating formula is

$$\lambda(t) = N_0/\{N_0 + e(t)^2 \cdot [1 + \psi(t-1)^T P(t-1)\psi(t-1)]^{-1}\} \quad (18)$$

where N_0 is an arbitrary positive tuning parameter called the memory length, and $e(t)$ is the estimation error. A typical value of N_0 is 10^4 (Ydstie et al., 1985).

Perturbation signal

Much of the convergence theory for adaptive control is based on the requirement of persistent excitation. There is no guarantee that the feedback signal will be a persistently exciting input (Anderson and Johnstone, 1983). In fact, under some circumstances unique parameter estimates cannot be found. Identifiability conditions for least squares estimation have been developed by Ljung (1979), Gustavsson et al. (1977), and Soderström et al. (1975). Hence an external signal (a perturbation signal) must usually be provided in adaptive control. Parameter estimation will be successful (i.e., converging) only when the energy level of the input is above a certain threshold, so it is important to monitor the excitation level (Peterson and Narendra, 1982; Egardt, 1979a, b; Clary and Franklin, 1985), which must be larger than the level of unmeasured disturbances. Moreover, persistent excitation is required to ensure a stable controller (Kosut and Johnson, 1984). The conditions on persistent excitation are more severe as the model order is increased (Wittenmark and Åström, 1984). Clary and Franklin (1985) have shown that the degree of excitation determines the number of parameters that can be uniquely identified.

Box and MacGregor (1974, 1976) were among the first investigators to suggest the use of a perturbation signal for closed-loop estimation. A perturbation signal in the process input is necessary to provide reliable parameter estimates when the elements of P are large. Since it is undesirable to add a perturbation signal continually, it may be added only when it is necessary by making the amplitude of the perturbation a function of the size of the elements of P. As suggested by Vogel (1982), the size of the elements of P may be indicated by the trace of P. Two functions were employed successfully by Vogel. The first was a linear relationship between the perturbation amplitude and the trace of P. To prevent excessively large perturbations, the amplitude was limited to a maximum allowable value when the trace of P reached a specified value. The second function was simply a step function. The perturbation amplitude has a specified nonzero value if the trace of P is above a given value and is zero otherwise.

The type of perturbation signal selected by Vogel was a pseudorandom binary sequence (PRBS). The period of a PRBS signal should be longer than the significant duration of the process impulse response. Eveleigh (1967) has discussed the properties of a PRBS and the generation procedure for a PRBS. At each sampling interval the appropriate element of the PRBS is multiplied by the perturbation amplitude and then added to the process input. Actually, in the closed loop the PRBS may be added either directly to the controller output, which is the process input, or to the set point. While the PRBS given above has yielded satisfactory results in actual applications, other perturbation signals may also be chosen (sinusoidal forcing was suggested by Goodwin and Teoh, 1983).

Estimator start-up

During start-up of a self-tuner or estimator, it is desirable that satisfactory parameter estimates be generated before the self-tuning controller is actually used to implement the control action. For example, a commissioning period can be employed during which time parameter estimation occurs but the process is controlled by a conventional PI or PID controller with conservative controller settings (Wittenmark, 1973; Cameron and Seborg, 1983).

Alternatively, a base case controller that corresponds to "safe" values of the parameter vector $\hat{\theta}(0)$ can be employed. Small-magnitude test signals may be used initially to excite the system, thus speeding up the estimation procedure (which can be done batchwise). During start-up it may be advisable to operate the estimator until satisfactory parameter estimates have been obtained before commissioning the adaptive controller, where the control action is based on the parameter estimates. Once operational experience has been obtained, it may be desirable to use the $\hat{\theta}$ from recent operating data in order to accelerate convergence. If there is a large number of parameters in the model, certain parameters may be assumed fixed during the start-up phase to assist convergence of the parameter estimation algorithm [this is done by setting the corresponding elements in $P(0)$ equal to zero]. It is important not to overparameterize the system model to any great degree (Åström, 1982).

Other estimator diagnostics

In order to achieve a robust adaptive controller, it is important to decouple the estimator requirements from the controller implementation. In other words, it is not necessary to strictly tie the estimator output to the controller. A formal means of doing this might involve the use of two time scales (Goodwin and Teoh, 1983; Johnstone and Anderson, 1982); the parameters are updated at each sampling instant but the control law parameters are updated only every M samples. Generally, ad hoc modifications where both estimator and controller use the same sampling rate but the estimator information is used selectively have been more popular. If a process is near steady state, the estimator will probably not operate as desired, so in this case the controller will operate independently of the estimator. A similar situation may arise in the start-up phase of an adaptive controller if a commissioning period is used.

When a change in the process parameters occurs, yielding a large estimation error, the parameter estimates may fluctuate drastically for a few iterations. In this situation, it is important not to update the controller settings based on parameter estimates that are grossly in error, since a poorly tuned controller and possibly an unstable system may result. Updating the controller with erroneous parameter estimates can be avoided by applying some simple, easily implemented tests to the new parameter estimates at each sampling instant. If the new estimates do not pass the tests, the controller parameters should not be updated at that sampling instant.

As suggested by Vogel (1982), the gain of the transfer function model provides a reasonably good indication of the quality of the parameter estimates. If the process model gain is unrealistic, the parameter estimates are erroneous and the controller parameters should not be updated using those estimates. Thus, if the process model gain does not lie within a specified, acceptable range, the controller parameters should not be updated at that sampling instant. The acceptable process gain range may be specified by high and low limits. The upper limit guards against the controller's being updated with erroneous parameter estimates that represent an inconsistently high gain of the process. Thus, since the controller gain is typically inversely related to the process gain, the upper limit prevents sluggish performance resulting from an extremely low controller gain. Conversely, in the case of erroneous parameter estimates that produce an unreasonably low process gain, the lower gain limit protects against oscillatory performance due to an abnormally high controller gain. The lower gain limit also guards against an unstable system resulting from control action in the wrong direction due to a process model gain with the wrong sign.

The poles of the discrete time model should also be checked periodically. To guard against erroneous parameter estimates and ensure satisfactory performance from the controller, the location of the poles of the process model should be examined at each iteration; if they are not within the acceptable region, the controller parameters should not be updated. With a second-order model this test is computationally easy, requiring the solution of a quadratic equation.

Even if the new parameter estimates pass the above tests, they may still represent relatively large fluctuations from the previous estimates. Therefore, to prevent large, sudden changes in the control action, the controller output must be constrained. Vogel (1982) suggested passing the new parameters through a first-order filter

$$\hat{\theta}_c(t) = \rho\theta_c(t-1) + (1.0-\rho)\,\hat{\theta}(t) \tag{19}$$

where $\hat{\theta}(t)$ is the vector of current parameter estimates and $\hat{\theta}_c(t)$ is a vector of parameter estimates used by the adaptive controller. ρ is the filter factor with a value between zero and one.

An application of recursive parameter estimation

In later sections we review various applications of recursive parameter estimation in the context of adaptive control. Here a typical application is discussed, illustrating how a parameter estimator performs with an actual operating process. Vogel and Edgar (1982a) tested an adaptive control algorithm on a heat exchange system. They used covariance resetting with $\lambda = 1$. After a start-up period in which the parameters were estimated, they introduced an unmeasured load disturbance (change in the hot stream flow rate) into the exchanger. They used a first-order model with $n = 1$, $m = 3$, and $k = 0$. Figure 2 shows the estimation error, which exhibits a large jump when the sustained disturbance enters (at $t = 2.6$ min). Since the operating conditions are now changed, the parameter estimates also change significantly; the bias term (d in Eq. 1) and the coefficients in $B(q^{-1})$ are shown in Figure 3. Note that b_1 and b_3 return close to their original values, while d and b_2 move to new values. The estimator used in this experimental application performed successfully; for more details, see Vogel (1982).

Conclusions and recommendations

While the development of on-line estimation algorithms is still an active research area, such algorithms have been successfully implemented in conjunction with adaptive control. Based on the results to date, there are several conclusions that can be drawn about the features of a successful estimation scheme:

1. The recursive least squares (RLS) method is the most popular estimation technique and appears to exhibit rapid convergence when properly applied. Extended least squares (ELS) using pseudolinear regression seems to be a satisfactory way to treat the non-Gaussian noise case, although many studies have found that the parameters in the C polynomial in Eq. 1 do not need to be estimated.
2. Either a variable forgetting factor and/or covariance resetting is required to keep the estimator running properly. A constant forgetting factor has many drawbacks. Monitoring the estimation error is a suitable means to decide when the covariance ought to be reset or how the forgetting factor should be adjusted.
3. A perturbation signal is required in order to achieve convergence in RLS, especially for higher order models. The perturbation signal should be large enough so that it is not masked by noise or unmeasured disturbances. When the control system is operating satisfactorily, a perturbation signal does not have to be used.
4. Control action based on the estimator may be ignored during the commissioning period or for other unusual operating conditions. Parameters may also be left unchanged when unmeasured disturbances enter the system, until there is time to analyze the new dynamic data. A successful adaptive control system does not necessarily operate the estimator at all times.
5. System diagnostics using physical limits and other considerations are very helpful in ensuring that the parameter estimates do not drift into unacceptable regions (see the earlier section on other estimator diagnostics).

Multivariable recursive estimation

Extension of the results given for single input–single output (SISO) models presented earlier to multiple input–multiple output (MIMO) systems is considered here. A discrete-time, transfer function model with an arbitrary number of inputs and outputs provides a convenient representation for adaptive control:

$$y(t) = G(q^{-1})\,u(t) + G_\xi(q^{-1})\,\xi(t) \tag{20}$$

where

$G(q^{-1}), G_\xi(q^{-1})$ = matrices of transfer functions involving the shift operator
$u(t), \xi(t)$ = input vectors
$y(t)$ = output vector

Each element of $G(q^{-1})$ has the same form as the SISO model given in Eq. 2. Thus, the $G_{ij}(q^{-1})$ element is

$$G_{ij}(q^{-1}) = \frac{x_{ij}(q^{-1})}{u_j(q^{-1})} = \frac{B_{ij}(q^{-1})}{A_{ij}(q^{-1})} \tag{21}$$

where

$$A_{ij}(q^{-1}) = \delta_{ij} - a_1^{ij} q^{-1} - a_2^{ij} q^{-2} - \cdots - a_n^{ij} q^{-n}$$

$$B_{ij}(q^{-1}) = b_1^{ij} q^{-1} + b_2^{ij} q^{-2} + \cdots + b_n^{ij} q^{-n}$$

n = order of the ijth element

$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$, Kronecker delta

$x_{ij}(q^{-1})$ = output from $G_{ij}(q^{-1})$

For N inputs the ith element of $y(q^{-1})$ when the noise model is neglected $[G_\xi(q^{-1}) = 0]$ is

$$y_i(q^{-1}) = \sum_{j=1}^{N} x_{ij}(q^{-1}) \tag{22}$$

The time delay is not explicitly included in Eq. 22 since different input-output combinations may have different time delays.

With the model given by Eq. 21, all the parameters may be estimated from the input and output data. Dahlqvist (1979) suggested that models of the form given by Eq. 21 may be simplified by neglecting the effects of the outputs on each other. To achieve this structure, $A(q^{-1})$ in Eq. 21 can be specified as a diagonal matrix of polynomials. However, in general $A(q^{-1})$ is not required to be diagonal.

To illustrate the form of the model in Eq. 22, consider a two-input, two-output system modeled by second-order polynomials ($n = 2$ for all i and j). In the time domain, the first element of the output vector at time t, $y_1(t)$, is related to past inputs and outputs as follows.

$$\begin{aligned} y_1(t) = {} & a_1^{11} y_1(t-1) + a_2^{11} y_1(t-2) \\ & + a_1^{12} y_2(t-1) + a_2^{12} y_2(t-2) + b_1^{11} u_1(t-1) \\ & + b_2^{11} u_1(t-2) + b_1^{12} u_2(t-1) + b_2^{12} u_2(t-2) \end{aligned} \tag{23}$$

$y_2(t)$ is given in a similar manner. Assume that $A(q^{-1})$ is a diagonal matrix so that the effects of the outputs on each other are neglected ($A_{ij} = 0$ for $i \neq j$). The corresponding time domain model for $y_1(t)$ in terms of the past inputs and outputs is:

$$\begin{aligned} y_1(t) = {} & a_1^{11} y_1(t-1) + a_2^{11} y_1(t-2) + b_1^{11} u_1(t-1) \\ & + b_2^{11} u_1(t-2) + b_1^{12} u_2(t-1) + b_2^{12} u_2(t-2) \end{aligned} \tag{24}$$

The expression for $y_2(t)$ is written in a similar manner.

Note that Eq. 24 is in the form of a multiple input–single output model where all parameters are unknown and must be estimated. If a bias term is added for the unknown disturbance, the model can be written for N inputs and N outputs as

$$y_i(t) = \psi_i^T(t-1)\,\theta_i(t-1) \tag{25}$$

where

$$\psi_i^T(t-1) = [y_i(t-1), \ldots, y_i(t-n), \; u_1(t-1), \ldots u_N(t-m), 1]$$

$$\theta_i^T(t-1) = [a_1^{i1}, \ldots, a_n^{iN}, b_1^{i}1, \ldots, b_m^{iN}, d_i]$$

Each parameter vector θ_i is estimated using the algorithm presented earlier; for each θ_i there will be a covariance matrix P_i.

While the above equations are fairly simple in form for parameter estimation, they present some difficulties in extending the SISO adaptive results to MIMO systems. Specifically, the representation of the time-delay terms in a multivariable system plays an important role in the design of adaptive algorithms for such processes (Elliott and Wolovich, 1984). The interactor matrix (Wolovich and Falb, 1976) is the most appropriate multivariable generalization of a SISO delay term for discrete systems and is required for the design of one-step-ahead predictive controllers. The interactor matrix is in a canonical form that is a product of a lower triangular unimodular matrix (with diagonal elements set equal to one) with a diagonal matrix that includes the time delay terms. While this approach can be cumbersome, it can be simplified for some special cases, as discussed by Dugard et al. (1984) and Elliott and Wolovich (1984). The interactor matrix ensures the use of the minimum order of predictors for multivariable systems with different time delays in each element. Such a characterization is important in formulating the MIMO model for parameter identification and control.

As in the SISO case, addition of perturbation signals to the inputs of the multivariable process is necessary to obtain satisfactory parameter estimates when the elements of P_i are large. For the multivariable parameter estimation, it is also important that the perturbation signals added to each input be uncorrelated in order to ensure reliable parameter estimates. For a two input–two output system, Gauthier and Landau (1978) suggested using a PRBS as one perturbation and the same PRBS delayed by half its length as the other perturbation.

To avoid unnecessary use of the PRBS signals, their amplitudes can be determined as functions of the size of the elements of P_i. Since each output is generally affected by all of the process inputs, the PRBS signals should be added to *all* of the process inputs whenever the elements of any of the P_i become large, as measured by the trace. Diagnostics should be applied to the new parameter estimates at each sampling instant before updating the process model parameters in the multivariable controller. These procedures are analogous to those described earlier for SISO systems. It should be emphasized that in the absence of a perturbation signal, adaptation of a MIMO model is exceedingly more difficult than adaptation of a SISO model.

Design Methods Based on Quadratic Cost Functions

In this section, we consider self-tuning control systems that are designed to minimize a quadratic cost function. The original idea is due to Kalman (1958). Twelve years later, Peterka (1970) revised the basic concept and extended the approach to stochastic systems. However, much of the current interest in self-tuning controllers was largely stimulated by the development of the self-tuning regulator (STR) by Åström and Wittenmark (1973) and the self-tuning controller (STC) by Clarke and Gawthrop (1975, 1979). These techniques will be described in the next two sections, followed by a discussion of the STC design parameters. Then a survey of experimental applications is presented. Succeeding sections are concerned with a number of important issues including: available theoretical results, potential operating problems, extensions to multivariable systems, and recent developments.

Self-tuning regulator

In the original STR of Åström and Wittenmark (1973), the feedback controller was designed to minimize the variance of the output variable, y. Thus the control objective was to minimize the quadratic cost function,

$$J_1 = \text{Var}\,(y) \tag{26}$$

where Var (y) denotes the variance of y. Minimum variance control has not been widely used in the process industries. However, it is a logical choice for quality control problems in applications where stochastic disturbances are important (e.g., papermaking). By reducing the output variance, the set point can be moved closer to a limiting constraint (Åström, 1980a; Clarke, 1981a).

Consider the process model in Eq. 2 with $n = m$. This assignment can be made without loss of generality, since any trailing coefficients in the A or B polynomials can always be set equal to zero if necessary. We also assume all of the roots of the B and C polynomials to be inside the unit circle of the complex plane, i.e., they all are located in the stable region.

The minimum variance control law is then given by (Åström, 1970):

$$u(t) = -\frac{F(q^{-1})}{B(q^{-1})\,E(q^{-1})}\,y(t) \tag{27}$$

where E and F are polynomials of order $k - 1$ and $n - 1$, respectively, with $e_0 = 1$.

These polynomials can be uniquely determined from the identity,

$$C(q^{-1}) = A(q^{-1})\,E(q^{-1}) + q^{-k}\,F(q^{-1}) \tag{28}$$

Combining Eqs. 2, 27, and 28 gives the closed-loop relation:

$$y(t) = E(q^{-1})\xi(t) = \xi\,(t) + e_1\xi(t-1) + \cdots + e_{k-1}\,\xi(t-k+1) \tag{29}$$

Thus the output is merely a moving average of the k previous random disturbances and the output variance is

$$\text{Var}\,(y) = \sigma^2\,[1 + e_1^2 + \cdots e_{k-1}^2] \tag{30}$$

where $\sigma = \text{Var}\,(\xi)$. Note that the variance increases with time delay, k.

The minimum variance control law can be interpreted as consisting of an optimal k-step-ahead predictor and a controller that attempts to set this prediction each to zero (Åström, 1970; Harris et al., 1982). In particular, the optimal k-step prediction, $y^*(t + k)$, based on current and past values of y and u up to time t, is given by (Åström, 1970):

$$y^*(t+k) = \frac{F(q^{-1})\,y(t) + B(q^{-1})\,E(q^{-1})\,u(t)}{C(q^{-1})} \tag{31}$$

The minimum variance control law in Eq. 27 can then be obtained by setting $y^*(t + k) = 0$. Note that $y(t + k)$ is the first value of y that can be affected by $u(t)$ due to time delay k in the process model of Eq. 2.

The relationship between the minimum variance controller and standard digital controllers such as the Smith predictor and Dahlin's controller has been evaluated by Palmor and Shinnar (1979) and Harris et al. (1982). In particular, they show that the minimum variance control law can be interpreted as a PID control law that contains a memory of previous control actions. The additional terms provide a form of time delay compensation.

Theoretically, the minimum variance control law in Eq. 27 is valid for any B polynomial. However, if B has any roots that lie on or outside the unit circle, the minimum variance controller is extremely sensitive to variations in the model parameters (Åström, 1970). Thus it should not be used in these situations. As indicated in the earlier sections on on-line parameter estimation, the sampled-data version of a continuous system can have a B polynomial with a zero outside the unit circle due to rapid sampling, or a time delay that is not an integer multiple of the sampling period (Gawthrop, 1980b; Åström et al., 1984).

Next, we consider an adaptive version of the minimum variance controller. A straightforward approach would be to estimate the A, B, and C polynomials on-line and then substitute these parameter estimates, $\hat{A}$, $\hat{B}$, and $\hat{C}$ for the unknown parameters in Eqs. 27 and 29. This adaptive control law would have the form

$$u(t) = -\frac{F(q^{-1})}{\hat{B}(q^{-1})E(q^{-1})}\,y(t) \tag{32}$$

where $E(q^{-1})$ and $F(q^{-1})$ are uniquely determined from the identity

$$\hat{C}(q^{-1}) = \hat{A}(q^{-1})\,E(q^{-1}) + q^{-k}F(q^{-1}) \tag{33}$$

by comparing coefficients of powers in q^{-1}. A disadvantage of this formulation is that the identity in Eq. 28 has to be solved on-line at each sampling instant in order to calculate $E(q^{-1})$ and $F(q^{-1})$. One way of avoiding this problem is to base the minimum variance control calculations on a predictive model that can be derived by combining Eqs. 2 and 33. This approach was used by Åström and Wittenmark (1973) in deriving their self-tuning regulator.

The predictive model for the STR has the form

$$y(t+k) + \alpha_1 y(t) + \cdots + \alpha_n y(t-n+1) = \beta_0[u(t) + \beta_1 u(t-1) + \cdots + \beta_\ell u(t-\ell)] + \epsilon(t+k) \tag{34}$$

where $\ell = n + k - 1$ and the random disturbance $\epsilon(t)$ is a moving average of order $k - 1$ of $\xi(t)$. The α_i and β_i parameters can be calculated from the a_i and b_i parameters in Eq. 1 by assuming that $C(q^{-1}) = 1$ and using the identity in Eq. 28. This assumption concerning $C(q^{-1})$ simplifies the subsequent parameter estimation problem since recursive least squares can be used; for $C(q^{-1}) \neq 1$, other techniques such as extended least squares must be employed.

Equation 34 is referred to as a predictive model because it allows the future output, $y(t + k)$, to be predicted from the current output $y(t)$, the current input $u(t)$, and previous values of y and u:

$$y^*(t+k) = -\alpha_1 y(t) - \cdots - \alpha_n y(t-n-1) + \beta_0[u(t) + \beta_1 u(t-1) + \cdots + \beta_\ell u(t-\ell)] \quad (35)$$

Equation 35 was obtained from Eq. 34 by replacing $y(t+k)$ by its predicted value $y^*(t+k)$, and setting the future disturbance, $\epsilon(t+k)$, equal to its mean value of zero. Note that Eq. 35 is a special case of Eq. 31 that can be obtained by setting $C(q^{-1}) = 1$ and expressing polynomials F and BE in terms of $\{\alpha_i\}$ and $\{\beta_i\}$.

The minimum variance control law can be derived by specifying $\hat{y}(t+k) = 0$ rearranging Eq. 35 to give

$$u(t) = \frac{1}{\beta_0}[\alpha_1 y(t) + \cdots + \alpha_n y(t-n+1)] - \beta_1 u(t-1) - \cdots - \beta_\ell u(t-\ell) \quad (36)$$

The control law in Eq. 36 is the minimum variance control law for both the predictive model in Eq. 35 and the original model in Eq. 2. Thus it is mathematically equivalent to the minimum variance control law in Eq. 27.

The self-tuning regulator is merely an adaptive version of the minimum variance controller in Eq. 36. Thus replacing the unknown parameters α_i and β_i by their estimates, $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$, gives the self-tuning regulator,

$$u(t) = \frac{1}{\hat{\beta}_0(t)}[\hat{\alpha}_1(t) y(t) + \cdots + \hat{\alpha}_n(t) y(t-n+1)] - \hat{\beta}_1(t) u(t-1) - \cdots - \hat{\beta}_\ell(t) u(t-\ell) \quad (37)$$

In summary, at each sampling instant the following calculations are performed:

1. Parameter estimates $\hat{\alpha}_i(t)$ and $\hat{\beta}_i(t)$ are updated from input/output data using recursive least squares.
2. The current control action, $u(t)$, is calculated from Eq. 37.

The STR in Eq. 36 has no provision for nonzero set points, integral action, or feedforward control. However, all of these features can be included (Wittenmark, 1973). In the original STR, the value of β_0 was assumed to be a known constant rather than an unknown parameter that had to be estimated. This ad hoc assumption was introduced as an attempt to avoid potential identifiability problems associated with closed-loop identification (Åström and Wittenmark, 1973). However, it may be difficult to select a suitable value of β_0 for a poorly understood process.

The self-tuning regulator has been used successfully in a number of experimental applications for both full-scale production plants and pilot-plant facilities (Åström et al., 1977; Parks et al., 1980). But simulation and experimental studies have demonstrated that the STR has a number of disadvantages:

1. It can be difficult to tune on-line.
2. Unknown or time-varying time delays can result in poor, even unstable, performance.
3. It is not directly applicable to nonminimum phase systems where B has a zero outside the unit circle.

The STR is difficult to tune on-line because it lacks a convenient tuning parameter. Model parameter β_0 is of some utility since it acts somewhat like the reciprocal of a controller gain (cf. Eq. 36). However, in some situations where the STR results in oscillatory responses and too vigorous control action, the only practical solution is to increase the sampling period. Unfortunately, the sampling period is not a convenient tuning parameter for many computer control software packages.

In the next section, we describe a generalization of the STR that circumvents most of these difficulties, the self-tuning controller of Clarke and Gawthrop (1975, 1979).

Self-tuning controller

The self-tuning controller (STC) can be derived in several different but equivalent ways (Clarke and Gawthrop, 1975, 1979; Clarke, 1981a, 1984). Here, we present an informal derivation due to Clarke (1981a). The starting point for the analysis is the discrete-time, stochastic model in Eq. 2 with $m = n$ and $d = 0$. The STC is expressed in terms of an auxilliary output ϕ defined by,

$$\phi(t) = P(q^{-1})y(t) + Q(q^{-1})u(t-k) - R(q^{-1})y_r(t) \quad (38)$$

where y_r is the set point and P, Q, and R are user-specified transfer functions of the form:

$$P(q^{-1}) = \frac{P_N(q^{-1})}{P_D(q^{-1})} \quad (39)$$

Guidelines for the selection of P, Q, and R will be discussed in the next section.

The control law used in the STC is designed to minimize the variance of auxiliary output ϕ. This control law, which is referred to as a generalized-minimum variance control law or predictive control law, has the form (Clarke, 1981a):

$$u(t) = \frac{Ry_r(t) - \phi_y^*(t+k)}{Q} \quad (40)$$

where $\phi_y^*(t+k)$ is the least squares prediction of $\phi(t+k) = P(q^{-1})\,y(t)$ made at time t. This optimal prediction is given by

$$\phi_y^*(t+k) = \frac{Fy_f(t) + Gu(t)}{C} \quad (41)$$

where $y_f = y/P_d$ and $G = EB$. Polynomials E and F are uniquely determined from the identity,

$$CP = AE + z^{-k}F/P_d \quad (42)$$

Polynomial F has a degree of $n-1$ plus the degree of P_D. The degree of polynomial E is $k-1$, which means that polynomial G in Eq. 41 has a degree of $n+k-1$. The degrees of F and G are important, since the coefficients of F and G are estimated on-line in the STC.

The following expression for the closed-loop system response can be derived (Clarke and Gawthrop, 1979) by combining Eqs.

2, 40, 41, and 42:

$$y(t) = \frac{EB + QC}{PB + QA}\xi(t) + \frac{q^{-k}BR}{PB + QA}y_r(t) \qquad (43)$$

Thus the characteristic equation is given by

$$PB + QA = 0 \qquad (44)$$

Note that the time delay term q^{-k} does not appear in the characteristic equation. Thus the predictive control law in Eq. 40 provides time-delay compensation via the $\phi_y^*(t + k)$ term. Other interpretations of this control law have been suggested by Gawthrop (1977) and Clarke (1981a).

The self-tuning controller of Clarke and Gawthrop (1975, 1979) is obtained by replacing ϕ_y^* in Eq. 40 by an estimate, $\hat{\phi}_y^*$:

$$u(t) = \frac{Ry_r(t) - \hat{\phi}_y^*(t + k)}{Q} \qquad (45)$$

where $\hat{\phi}_y^*(t + k)$ is given by

$$\hat{C}\,\hat{\phi}_y^*(t + k) = \hat{F}y_f(t) + \hat{G}u(t) \qquad (46)$$

Parameter estimates $\hat{F}$, $\hat{G}$, and $\hat{C}$ are calculated on-line using the predictive model in Eq. 41 and a recursive technique such as extended least squares (Clarke, 1981a). If it is assumed that $C(q^{-1}) = 1$, then recursive least squares can be used instead of extended least squares.

Although the predictive control law in Eq. 40 was designed to minimize the variance of ϕ, Clarke (1981a) has shown that it also minimizes a quadratic cost function J_2,

$$J_2 = \epsilon\{[P(q^{-1})y(t + k) - R(q^{-1})y_r(t)]^2 + \mu[Q(q^{-1})u(t)]^2\} \qquad (47)$$

where $\mu = g_0/q_0$ and ϵ denotes the expectation operation. The expectation is conditioned with respect to data up to time t (MacGregor, 1977). To illustrate the utility of cost function J_2, we will consider a few specific examples. If P, Q, and R are chosen to be scalars: $P = R = 1$ and $Q = \lambda'$, then J_2 reduces to

$$J_2 = \epsilon\{[y(t + k) - y_r(t)]^2 + \lambda'[u(t)]^2\} \qquad (48)$$

This type of quadratic cost function has been widely used in optimal control theory (Ray, 1981). Scalar λ' provides a convenient tuning factor that can be used to make the control action more or less vigorous. In particular, it can be used to trade off the variance of the error signal, $e = y - y_r$, against the variance of u (Clarke, 1981a).

As a second example, suppose $P = R = 1$ and $Q = \lambda'(1 - q^{-1})$. Then the cost function becomes

$$J_2 = \epsilon\{[y(t + k) - y_r(t)]^2 + \lambda'[u(t) - u(t - 1)]^2\} \qquad (49)$$

By penalizing the incremental change in u, the resulting controller contains integral action in analogy to the well-known situation for optimal control (Hammerström and Gros, 1980; Wong and Seborg, 1985b). For the situation where $P = R = 1$, $Q = 0$, and $y_r = 0$, then $J_2 = \epsilon[y^2(t + k)]$ and the resulting controller is the minimum variance controller of Eq. 36.

The generalized minimum variance control law in Eq. 40 provides several important advantages over the minimum variance control law in Eq. 36:

1. It is more easily tuned.
2. It can be applied to nonminimum phase plants.

The standard STC, like the STR, may perform poorly if the process time delay k is unknown or time-varying. Recently, Chien et al. (1984a) have proposed a modified version of the STC that is less sensitive to unknown or varying time delays. Clarke (1982) has compared the performance of the STC and pole-placement design methods for systems with variable time delays.

Selection of design parameters

We have seen that self-tuning control systems consist of a combination of a feedback control strategy and an on-line parameter estimation scheme. Consequently, the design of a self-tuner includes the specification of three types of design parameters: model parameters, control parameters, and estimation parameters. A detailed discussion of each design parameter and its influence on control system performance is beyond the scope of this paper. Instead, we will present a brief overview of the design parameter selection for a popular self-tuner, the STC of Clarke and Gawthrop. For more detailed analyses of the design parameters for various self-tuners, see the papers by Åström (1980a, b), Clarke (1981b), Wellstead and Zanker (1982), Isermann (1982), and Wittenmark and Åström (1984).

As an illustrative example, we consider the STC based on the generalized minimum variance controller in Eq. 40 and extended least squares estimation.

Model Parameters. Since the STC is based on the discrete-time model in Eq. 2, four model parameters must be specified *a priori:* model orders n and m, time delay k, and sampling period Δt. Key considerations affecting the selection of these model parameters have been presented in the section on linear difference equation models.

Control Parameters. The controller design parameters for the STC are the P, Q, and R transfer functions in Eqs. 38 and 47. Some guidelines concerning the choice of these parameters will be summarized here. From the closed-loop relation in Eq. 43 it is clear that for $Q = 0$ and $R = 1$, the closed-loop transfer function for set-point changes is $M(q^{-1}) = 1/P(q^{-1})$. Thus for this situation, P can be specified to give a desired closed-loop response for set-point changes (Clarke, 1981a). Typically, $M(q^{-1})$ is selected to be a low-order transfer function (plus zero-order hold) whose dynamics are somewhat faster than the open-loop system.

Transfer function $R(q^{-1})$ can be selected to provide set-point filtering and thus tailor the set-point response without affecting the load response. For example, if step changes in set point are undesirable, more gradual changes can be introduced by specifying $R(q^{-1}) = r/(1 - r_1q^{-1})$. This first-order filter converts a step change in y_r to an exponential change in the filtered set point and produces less overshoot in the output variable. Parameter r_1 determines how fast the exponential response will be; parameter r should be adjusted so that $P(q^{-1})$ and $R(q^{-1})$ have the same steady-state gains. Otherwise offset will occur. (The steady-state gain for these transfer functions can be obtained by setting $q = 1$.)

Polynomial $Q(q^{-1})$ can be used to reduce the control effort by penalizing the control variable u. A typical choice is $Q(q^{-1}) = \lambda'(1 - q^{-1})$ or $Q(q^{-1}) = \lambda'(1 - q^{-1})/(1 - \alpha q^{-1})$ where α is a constant. Increasing λ' tends to make the control action less vigorous, while the $(1 - q^{-1})$ term provides integral action to eliminate offsets after load and set-point changes. A number of alternative methods have been proposed for eliminating offset, including: estimation of bias d in the process model of Eq. 2, modification of the process model, or addition of an integrator in either an inner or outer feedback loop (Clarke and Gawthrop, 1979; Åström, 1980a; Modén, 1981b; Allidina and Hughes, 1982). Clarke et al. (1983) have provided an in-depth discussion of alternative approaches and have proposed a promising new approach based on a modified predictor.

Toivonen (1983c) and Leong et al. (1984) have suggested automatic adjustment of $Q(q^{-1}) = \lambda'$ to achieve desired closed-loop performance.

Estimation Parameters. In order to use a recursive estimation scheme such as extended least squares, several parameters must be specified: forgetting factor λ, initial parameter estimate $\hat{\theta}(0)$, and initial covariance matrix $P(0)$. Guidelines for the selection of these parameters have been presented in earlier sections.

Experimental applications

The theoretical development of self-tuning control systems during the past decade was accompanied by a wide variety of experimental applications. In fact, early industrial applications in Sweden (Åström et al., 1977) were reported within one year after the landmark STR paper by Åström and Wittenmark (1973) was published. Table 2 summarizes experimental applications of self-tuning controllers to process control problems. This summary includes only those experimental applications in which the self-tuner was designed to minimize a quadratic cost function. Applications of other design methods based on pole placement and closed-loop stability will be discussed in later sections.

In Table 2 a distinction is made between applications to laboratory-scale equipment and those that involved full-scale industrial plants. Most of the reported industrial applications consisted of short-term experiments to demonstrate the feasibility of self-tuning. However, at least a few of the industrial applications have been in operation for several years (Dumont and Bélanger, 1978, 1981; Westerlund et al., 1980). Also, Hodgson (1982) reported that a self-tuning controller implemented on a portable microprocessor provided several months of troublefree operation in applications involving three large batch reactors for latex polymerization.

It is noteworthy that the majority of the industrial applications of self-tuning controllers have occurred in Europe and Canada. By contrast, only a few applications have been reported for industrial plants in the United States (Fjeld and Wilhelm, 1981; Hoopes et al., 1983; Piovoso and Williams, 1984). Since general-purpose self-tuners are now commercially available, widespread industrial application of self-tuners in future years is anticipated.

Theoretical results

A number of theoretical studies have considered important issues associated with self-tuners including the self-tuning property, asymptotic convergence, and robustness. These analyses are quite difficult and abstract since the differential equations

Table 2. Experimental Process Control Applications of Adaptive Control Systems Based on Quadratic Cost Functions

Application	References
Absorption/Desorption Plants	Fortescue et al. (1981); Kershenbaum and Fortescue (1981); Ydstie et al. (1985); Hiram and Kershenbaum (1985)
Cement Raw Material Blending	Talabér et al. (1977); Keviczky et al. (1978); *Westerlund et al. (1980)
Chemical Reactors	*Ahlberg and Cheyne (1976); Harris et al. (1978); Buchholt et al. (1979); Clarke and Gawthrop (1981); Yang et al. (1981); Hallager and Jørgensen (1981, 1983a,b); *Hodgson (1982); *Hodgson and Clarke (1982); McDermott (1984); *Bengtsson and Egardt (1984); Hallager et al. (1984); *McAlpine (1985)
Chip Refiner	*Dumont (1982)
Concentration-Flow Process	Koivo et al. (1981)
Cryrostat	*Hodgson (1982)
Diesel Engine	Wellstead and Zanker (1981)
Digesters	*Cegrell and Hedqvist (1975); *Sastry (1977); Bélanger et al. (1983)
Distillation Columns	Sastry et al. (1977); Morris et al. (1977, 1980, 1981, 1982); Dahlqvist (1979, 1981); Lieuson et al. (1980); Clough and Lovece (1981); Chien et al. (1985b); Vagi et al. (1985)
Driers	*Modén and Nybrant (1980); *Modén (1981a); *Najim et al. (1982); *Tuffs and Clarke (1985)
Evaporator	Chang (1975); Buchholt and Kümmel (1981)
Extruder	Bezanson (1983)
Heat Exchangers and Heating Systems	Jensen and Hansel (1974); Kurz et al. (1980); Bergmann and Radke (1980); Clarke and Gawthrop (1981); Dexter (1981); Radke (1982a,b); de Keyser and van Cauwenberghe (1982); Schumann (1982); Cameron and Seborg (1983); *Hoopes et al. (1983); Radke (1982a,b); Radke and Isermann (1984); Lee et al. (1985); *Nesler (1985); *Graham and Dexter (1985)
Liquid Level	Gawthrop (1981); de Keyser and van Cauwenberghe (1982)
Nuclear Reactor	Allidina et al. (1981)
Ore Crushing	*Borisson and Syding (1976)
Paper Machines	*Borisson and Wittenmark (1974); *Cegrell and Hedqvist (1975); *d'Hulster et al. (1980); *Fjeld and Wilhelm (1981); *Fjeld (1982); *de Keyser and van Cauwenberghe (1982); Søeberg (1982)
pH Control	Buchholt and Kümmel (1979); Bergmann and Lachmann (1980); *Jacobs et al. (1980, 1985); Clarke and Gawthrop (1981); *Proudfoot et al. (1983, 1985); Gustafsson (1985); *Piovoso and Williams (1984)
Plastic Film	*Mäkilä and Syrjänen (1983)
Rolling Mill	*Bengtsson (1979); *Bengtsson and Egardt (1984)
Rotary Kiln	*Dumont and Bélanger (1978); *Mejnertsen and Williams (1982)
Furnace	de Keyser and van Cauwenberghe (1981); Haber et al. (1981); Kawata et al. (1984)

*Denotes experimental application to full-scale industrial plant.

that describe the closed-loop behavior of self-tuners are nonlinear, time-varying, and stochastic. Despite this inherent complexity, considerable progress has been made, especially for the ideal situation where the model structure is correct and the model parameters are constant.

Most of the self-tuners that have been proposed, including the STR and the STC, exhibit the self-tuning property. That is, the controller parameters eventually converge to the values that would be used if the actual process model parameters were known. A related concept, global convergence, describes the desirable situation where the tracking error, $e = y_r - y$, asymptotically approaches zero and the input $u(t)$ and output $y(t)$ are uniformly bounded. Goodwin et al. (1980, 1981) have proved global convergence for broad classes of both SISO and MIMO self-tuners. These papers also summarize earlier research on this topic. Osorio-Cordera and Mayne (1981) and Lozano (1982) have analyzed the convergence properties of STR algorithms with variable forgetting factors. Goodwin et al. (1983) have performed a similar analysis for a STR with covariance resetting. The exponential convergence of adaptive control algorithms has also received considerable attention (Anderson and Johnson, 1982).

In recent years there has been growing interest in analyzing the robustness characteristics of self-tuners. The term robustness refers to the performance of a self-tuner during realistic conditions where the assumed process model structure is incorrect, plant variations occur, unanticipated disturbances and plant constraints are present, and hardware limitations such as magnitude and rate limits must be considered. Numerous simulation and experimental studies have demonstrated that self-tuners can be made quite robust but that practical limits do exist. However, the theoretical results concerning robustness that are presently available are quite restricted (Gawthrop and Lim, 1982; Lim, 1982).

Potential operating problems

The theoretical benefits that can be provided by self-tuning control systems are readily apparent. Technical feasibility is not an issue since self-tuners can be easily implemented via microprocessors (Glattfelder et al., 1980; Clarke and Gawthrop, 1981). But in order to achieve widespread industrial acceptance it is essential that self-tuners be robust enough to cope with a wide range of real-world conditions. Potential operating problems associated with self-tuners and ways of avoiding them are described in the informative survey articles by Åström (1980b, 1983), Clarke (1981b), Clarke and Gawthrop (1979), Isermann (1982), Wellstead and Zanker (1982), and Wittenmark and Åström (1984), and in the papers by Bristol (1983), Yang and Lee (1983), Latawiec and Chyra (1983), and Clarke et al. (1983).

Many potential operating problems can be avoided or their effects greatly reduced by careful implementation of the estimator and by use of appropriate estimator diagnostics, as discussed in the earlier sections on operating problems of parameter estimation algorithms.

Multivariable control problems

In recent years there has been considerable research effort devoted to the development of self-tuners for processes with multiple inputs and multiple outputs, i.e., for multivariable control problems. A typical starting point in these analyses is the multivariable generalization of Eq. 2:

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t-k) + C(q^{-1})\, \xi(t) \qquad (50)$$

where y is the output vector, u is the input vector, and ξ is the random noise vector. All three vectors are n-dimensional. Matrices $A(q^{-1})$, $B(q^{-1})$, and $C(q^{-1})$ are all $n \times n$ matrices whose elements are polynomials in the shift operator q^{-1}. For example,

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \cdots + B_n q^{-n} \qquad (51)$$

It is usually assumed that $A_0 = C_0 = I$, B_0 is nonsingular, and that the roots of det $C(q^{-1})$ lie inside of the unit circle in the complex plane (Koivo, 1980).

The process model in Eq. 50 implies that there is a time delay of at least k sampling periods associated with each input-output pair. Longer delays can be included in the model by specifying that selected elements of the $\{B_i\}$ matrices are zero. For example, if $k = 1$ and a time delay of two sampling periods is associated with the response of y_2 to u_1, then the (2, 1) elements of B_0 and B_1 should be zero.

Multivariable versions of the STR have been developed for problems where B_0 is nonsingular and where det $B(q^{-1})$ has all its roots inside the unit circle (Borrison, 1975, 1979; Keviczky and Hetthésey, 1977; Keviczky et al, 1978; Bayoumi and El-Bagoury, 1979). Both requirements are quite restrictive; B_0 is often nonsingular due to the presence of different time delays for various input-output pairs. For nonminimum-phase systems, det $B(q^{-1})$ will have at least one root located outside of the unit circle. As discussed earlier, nonminimum-phase systems can occur when rapid sampling is used or if the time delay is not an integer multiple of the sampling period (Åström et al., 1984; Clarke, 1981a). If the minimum variance controller results in excessive control action, the control action can be made less vigorous by introducing a slow pole in the closed-loop system. The resulting scheme has been referred to as detuned minimum variance control (Wellstead and Zanker, 1982).

Multivariable extensions of the Clarke-Gawthrop self-tuning controller have been developed by several research groups (Koivo, 1980; Lu and Yuan, 1980; Keviczky and Kumar, 1981; Bayoumi et al., 1981; Wong and Bayoumi, 1981; Grimble and Moir, 1983; Chien et al., 1983, 1984b, 1985a, b). Like the original Clarke-Gawthrop STC, these methods allow cost function penalties to be placed on the manipulated input u and are applicable to nonminimum phase systems. Although early papers were based on the restrictive assumption that each input-output pair had the same time delay, more recently developed design methods can accommodate arbitrary time delays for each pair (Morris et al., 1981, 1982; Tanttu and Koivo, 1983; Chien et al., 1984a, 1985a; Dugard et al., 1984). A few design methods permit different numbers of inputs and outputs (Wong and Bayoumi, 1981; Favier and Hassani, 1982; Grimble et al., 1982; Toivonen, 1983b) or a different sampling rate for each output variable (Morris et al., 1981, 1982; Chien et al., 1983; Costin and Buchner, 1983). Some multivariable self-tuners are applicable to plants that are either open-loop and unstable or nonminimum-phase (McDermott and Mellichamp, 1983; Grimble and Moir, 1983). McDermott and Mellichamp (1984a) and Jun et al. (1985) select the weighting matrices in the quadratic cost function so as to assign closed-loop poles and to achieve approximate decoupling. Self-tuners that include classical decoupling controllers have also been proposed (Gawthrop, 1983; Chien et al., 1984b) and experimentally evaluated (Chien et al., 1985b).

Multivariable self-tuners designed using a linear-quadratic-Gaussian approach have recently been reviewed by Grimble (1984a, b). They typically have increased on-line computational requirements in comparison with the STR and STC approaches (Peterka and Åström, 1973; Åström, 1980a; Lam, 1980; Hallager and Jørgensen, 1981; Grimble et al., 1982; Grimble, 1984a, b), but the computational load can be reduced by solving the Ricatti equation by performing only a single iteration at each sampling instant.

Experimental applications of multivariable self-tuners have been reported for several industrial plants (Borisson, 1975; Talabér et al., 1977; Keviczky et al., 1978; Najim et al., 1982), as well as laboratory-scale distillation columns (Morris et al., 1980, 1981, 1982; Dahlqvist 1979, 1981; Chien et al., 1985b; Vagi et al., 1985) and chemical reactors (Buchholt et al., 1979; Hallager and Jørgensen, 1981, 1983a, b; McDermott, 1984).

Some recent developments

So far in this section we have emphasized the STR of Åström and Wittenmark and the STC of Clarke and Gawthrop. We will now briefly summarize some extensions and related developments.

The STC of Clarke and Gawthrop (1975, 1979) is an implicit self-tuner which is designed to minimize the k-step-ahead performance index in Eq. 47. This performance index is referred to as a single-stage cost function because it is only concerned with what happens at a single sampling instant, k steps ahead. A number of recent studies have considered alternative cost functions. In analogy with the well-established linear-quadratic-Gaussian (LQG) theory of optimal control (Ray, 1981), self-tuners can be designed to minimize the following cost function (Lam, 1980; Grimble et al., 1982; Samson, 1982; Grimble, 1984a, b)

$$J_3 = \epsilon[e^T(t)Qe(t) + u^T(t)Ru(t)] \tag{52}$$

where for the multivariable control problem, e is the m-dimensional error vector, $e = y_r - y$, and u is the m-dimensional control vector. The expectation in Eq. 52 is unconditional, in contrast to the conditional expectation used in the standard Clarke-Gawthrop formulation of Eq. 47. A disadvantage of this approach is the increased computational requirements, which occur because the control law calculations typically involve either spectal factorization or the solution of a Riccati equation. Alternatively, one can use a receding horizon or extended horizon cost function that weighs y and u at N time steps rather than at a single time step. For example, Lam (1980) has performed a detailed analysis using the SISO cost function:

$$J_4 = \epsilon\left[\sum_{t}^{N+t-1} y_{t+k}^2 + \lambda' u_t^2\right] \tag{53}$$

where the expectation is conditional upon data up to time t. Related approaches have been considered by Greco et al. (1984), de Keyser and van Cauwenberghe (1981, 1982), Ydstie (1982, 1984a, b), Samson (1982), Lee and Lee (1985), and van Cauwenberghe and de Keyser (1985).

Most of the self-tuning controllers that are available are based on a discrete-time transfer function model of the process such as Eq. 2. However, several recent papers have employed other types of process models. Lam (1980), Warwick (1981, 1982) and Bezanson and Harris (1984) have recently proposed several explicit self-tuners based on state-space models. Adaptive versions of dynamic matrix control (Freedman and Bhatia, 1985) and internal model control (Svoronos, 1985) have also been proposed. Gawthrop (1980b, 1982b) has developed hybrid self-tuners that are based on continuous-time transfer function models. Special types of nonlinear models such as bilinear models (Svoronos et al., 1981), Hammerstein models (Anbumani et al., 1981; Lachman, 1982), and sector nonlinearities (Golden and Ydstie, 1985) have also been utilized. Agarwal and Seborg (1985) have recently extended the Clarke-Gawthrop formulation to a general class of nonlinear models. Nonparametric approaches to self-tuning have been developed by Wellstead and Zarrop (1983).

Two recent papers have proposed modifications to the Clarke-Gawthrop design procedure in order to reduce the variance of the input variable (Toivonen, 1983a; Bayoumi and Ballyns, 1983). Self-tuning control in the presence of input constraints has been considered by Mäkilä (1982) and Bezanson (1984).

Åström (1985) has recently proposed an expert system approach to coordinate alternative adaptive control strategies such as gain scheduling, self-tuning, and autotuning.

Design Methods Based on Pole Placement

As an alternative to controller design based on optimization of some performance index, Wellstead et al. (1979) and Åström and Wittenmark (1980) have suggested self-tuning controllers with the controller design based on pole placement. The presentation in this section differs from the preceding one in that in most cases we use an explicit approach to self-tuning. The model parameters are first estimated, followed by design of the controller assuming that these parameter estimates are correct. There are only a few cases where implicit self-tuners based on pole placement can be employed.

The pole placement controller has the general form shown in Figure 4:

$$\bar{F}(q^{-1})u(t) = \bar{H}(q^{-1})y_r(t) - \bar{G}(q^{-1})y(t) \tag{54}$$

where

$$\bar{F}(q^{-1}) = 1 + \bar{f}_1 q^{-1} + \cdots + \bar{f}_n q^{-n}$$
$$\bar{G}(q^{-1}) = \bar{g}_0 + \bar{g}_1 q^{-1} + \cdots + \bar{g}_n q^{-n}$$
$$\bar{H}(q^{-1}) = \bar{h}_0 + \bar{h}_1 q^{-1} + \cdots + \bar{h}_n q^{-n}$$

Given a process model of the form of Eq. 2, the closed-loop transfer function is given by

$$y(t) = \frac{\bar{F}C}{A\bar{F} + \bar{G}Bq^{-k}}\xi(t) + \frac{\bar{H}B}{A\bar{F} + \bar{G}Bq^{-k}}y_r(t) \tag{55}$$

where A, B, and C are defined in Eq. 2. $\bar{F}$ and $\bar{G}$ are selected so that the closed-loop poles are at the desired locations as specified by the polynomial $T(q^{-1})$, defined as

$$T(q^{-1}) = 1 + t_1 q^{-1} + \cdots + t_n q^{-n} \tag{56}$$

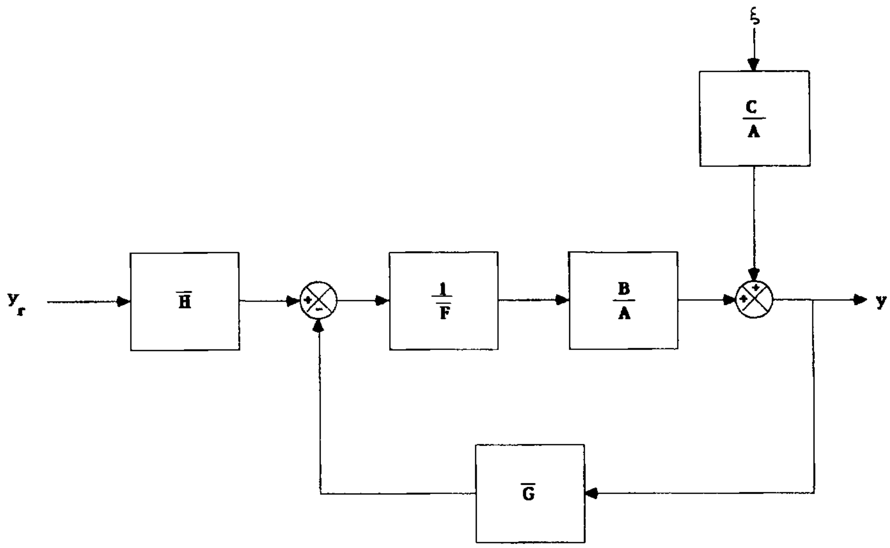

Figure 4. Block diagram for control law.
$\bar{F}, \bar{G}, \bar{H}$ in Eq. 54 for model
A, B, C in Eq. 2, $d = 0$.

The nth order polynomials $\bar{F}(q^{-1})$ and $\bar{G}(q^{-1})$ are determined by requiring that

$$A\bar{F} + \bar{G}Bq^{-k} = TC \tag{57}$$

which is known as the Diophantine equation. The parameters of $\bar{F}$ and $\bar{G}$ are obtained by equating like powers of q^{-1} in Eq. 57, which yields a set of simultaneous equations. With this selection for the controller, the closed-loop equation for the regulator case ($y_r = 0$) is

$$y(t) = \frac{\bar{F}(q^{-1})}{T(q^{-1})}\xi(t) \tag{58}$$

See Clarke (1981a) for a sample calculation of $\bar{F}$ and $\bar{G}$ for a simple process model.

An advantage of the pole placement controller is that it can be tuned by appropriate shifting of the closed-loop poles, which allows the controller actions to be limited as desired. In addition, the pole placement controller can be applied to nonminimum-phase processes. Although the controller shown above does not guarantee equality of the output and set point (i.e., no offset), integral action and set-point tracking features may be added, as discussed later. A disadvantage of the pole placement design approach is that the actual closed-loop performance is uncertain before the controller parameters are determined since, as shown in Eq. 58, the closed-loop zeros depend on the control law polynomials. Consequently, after the closed-loop poles are selected, the controller parameters must be calculated to determine the location of the closed-loop zeros and to verify the performance of the resulting controller.

Wellstead-Prager procedure

For the design procedure described above, implementation of the pole placement controller as an explicit self-tuning controller requires the solution of the set of $(n + k)$ simultaneous equations, Eq. 57, at each iteration. The complexity of this solution thus depends on the model order (n) and delay time (k). While certain conditions could cause the solution for the controller parameters to be ill-conditioned, this does not seem to be a frequently occurring problem. This self-tuning algorithm may also be formulated as an implicit method for one special case (Wellstead and Sanoff, 1981; Allidina and Hughes, 1982). However, the implicit algorithm may be unstable for nonminimum-phase processes if the closed-loop poles are not selected properly.

Like the self-tuning controllers discussed in the previous section, pole placement controllers provide dead time compensation. Wellstead et al. (1979) have suggested a modification to the process model of their self-tuning pole placement algorithm that allows the controller to adapt to an unknown or varying dead time without requiring an explicit estimate of the dead time. Specifically, k in the process model is selected to correspond to the minimum expected dead time k_1, and the numerator polynomial, $B(q^{-1})$, is extended by k_2 terms, where $k_1 + k_2$ is the maximum expected dead time. In this case, the dead time is modeled by leading coefficients of $B(q^{-1})$, which may be zero. If all coefficients in $B(q^{-1})$ are zero except for the last one (highest power of q^{-1}), this corresponds to a change in the dead time to $k_1 + k_2$. While overparameterization of $B(q^{-1})$ offers a solution to the problem of performance loss due to unknown or varying dead time, the complexity of the algorithm is increased with the dead time. For example, for a process with large dead time, the order of the controller polynomial $\bar{F}(q^{-1})$ is high, and in turn the number of simultaneous equations to be solved is large. As a result, the algorithm is not attractive for processes with large dead times (see the section on the Vogel-Edgar controller, below, for a solution to this problem).

The self-tuning regulator discussed previously is related to a pole placement controller. According to Eq. 44, the characteristic equation is given by the solution of a polynomial equation. The roots of $PB + QA$ can be shifted by using different values of Q. In the normal application of the STC, P is chosen to be 1.0 and Q is taken to be $\lambda'(1 - q^{-1})$. But as discussed by McDermott and Mellichamp (1983, 1984a), such assumptions about P

and Q are quite restrictive for complicated dynamic systems. They recommended that higher order models for P be selected. The solution of the simultaneous equations for the closed-loop poles can be done quickly at each time step and need not be carried out when there is little change in the model parameters.

The selection of the closed-loop poles is tied to the value of Δt employed; if Δt is changed, the pole also must be changed to maintain response quality. The sampling period can be adjusted on-line if necessary, followed by selection of the optimum closed-loop pole. Simulation results by McDermott and Mellichamp (1983) have shown that such a one-dimensional optimization is feasible to carry out on-line and does provide a noticeable improvement in load responses.

Åström-Wittenmark procedure

A more general design procedure for self-tuning controllers based on pole placement and zero cancellation has been presented by Åström and Wittenmark (1980). With their procedure, the desired controller is derived from specification of the servo performance, while the controller designs for the previously discussed self-tuning algorithms have been based on regulatory performance. The process model used for their algorithm is given by Eq. 1.

In Åström and Wittenmark's design procedure, the controller is designed by specifying the desired transfer function between the set point and output as

$$y(t) = \frac{B_m(q^{-1})q^{-k}}{A_m(q^{-1})} y_r(t) \tag{59}$$

In contrast to the previously discussed pole placement design method, this procedure places the closed-loop poles but also provides for cancellation of some or all of the zeros. Hence the closed-loop response is completely specified with no uncertainty. The general form for the controller is given by the feedback law, Eq. 54. In some cases, $\overline{H}(q^{-1}) = \overline{G}(q^{-1})$, and the control law of Eq. 54 can be written in the more common form given below.

$$u(t) = \frac{\overline{G}}{\overline{F}} [y_r(t) - y(t)] \tag{60}$$

With the controller of Eq. 65, the closed-loop transfer function is

$$\frac{y}{y_r} = \frac{\overline{H}Bq^{-k}}{A\overline{F} + \overline{G}Bq^{-k}} = \frac{B_m q^{-k}}{A_m} \tag{61}$$

where the righthand side is the desired closed-loop transfer function of Eq. 59. Note that the q^{-1} operator has been omitted as argument of the polynominals.

The controller design problem is to find $\overline{F}$, $\overline{G}$, and $\overline{H}$ to satisfy Eq. 61. Certain choices of the form of $\overline{F}$, $\overline{G}$, and $\overline{H}$ lead to a PID controller when the process model is second order (Tjokro and Shah, 1985). More generally, it is apparent from Eq. 61 that any factors of B that are not factors of B_m must be factors of $\overline{F}$, so they will cancel. Consequently, open-loop zeros that are not desired as closed-loop zeros must appear in $\overline{F}$. Conversely, open-loop zeros that are not desired as controller poles in $\overline{F}$ must appear in B_m. Therefore, factor B as

$$B = B^+B^- \tag{62}$$

where all of the zeros of B^+ are located within the unit circle and correspond to well-damped response modes. All of the zeros of B^- are outside the unit circle (unstable) or correspond to poorly damped (ringing) modes. With Eq. 62, the closed-loop transfer function of Eq. 61 becomes

$$\frac{\overline{H}B^+B^-q^{-k}}{A\overline{F} + \overline{G}B^+B^-q^{-k}} = \frac{B_m q^{-k}}{A_m} \tag{63}$$

In order to obtain a satisfactory controller, the zeros of B^- are not desired as controller poles in $\overline{F}$. Thus, it is clear from Eq. 63 that B^- must appear in B_m, and B_m is factored as

$$B_m = B_{m1}B^- \tag{64}$$

where B_m represents desired closed-loop zeros not included in B^-. In addition, Eq. 63 implies that B^+ must appear in $\overline{F}$, which is expressed as

$$\overline{F} = \overline{F}_1 B^+ \tag{65}$$

$\overline{F}_1$ corresponds to controller poles in addition to those of B^+. Substituting Eqs. 64 and 65 into Eq. 63 and canceling like terms, Eq. 63 becomes

$$\frac{\overline{H}}{A\overline{F} + \overline{G}B^-q^{-k}} = \frac{B_{m1}}{A_m} \tag{66}$$

Since the order of the polynomial A_m is normally less than the order of $(A\overline{F}_1 + \overline{G}B^-q^{-k})$, there are factors in Eq. 66 that cancel, and $\overline{H}$ may be factored as

$$\overline{H} = \overline{H}_1 B_{m1} \tag{67}$$

Thus, Eq. 66 becomes

$$A\overline{F}_1 + \overline{G}B^-q^{-k} = A_m\overline{H}_1 \tag{68}$$

$\overline{H}_1$ is specified by the user, and the controller design procedure then requires the solution of Eq. 68 for $\overline{F}_1$ and $\overline{G}$. As in previous cases, this may be done by equating coefficients of like powers of q^{-1}, which yields a set of simultaneous equations. The controller polynomials $\overline{F}$ and $\overline{H}$ are obtained from $\overline{F}_1$ and $\overline{H}_1$ through Eqs. 65 and 67. Åström and Wittenmark (1980) have discussed the selection of $\overline{H}_1$ and the orders of $\overline{F}_1$ and $\overline{G}$.

Implementation of the pole placement controller designed above as a self-tuning controller requires the following steps to be performed at each iteration:

1. Estimation of the process model parameters, Eq. 1.
2. Factorization of B (actually $\hat{B}$, the estimated model parameters) as in Eq. 62.
3. Solution of a set of simultaneous equations to obtain the controller parameters from Eq. 68.
4. Calculation of the control action from Eq. 54.

Note that this is an explicit self-tuning algorithm.

One disadvantage of this procedure is that factoring B as in Eq. 62 is essentially a spectral factorization, which is not desirable to perform each sampling interval as part of the self-tuning algorithm. Two special cases, which avoid the factorization, are given below.

Case 1. All Process Zeros Are Canceled. Here, all process zeros are canceled and no additional zeros are introduced. This means that in Eq. 62, $B^- = 1$ and $B_m = \overline{K}$, where $\overline{K}$ is a constant. Thus, with $B^- = 1$ and $B_m = \overline{K}$, Eqs. 64, 65, 66, and 68 are, respectively,

$$B_m = B_{m1} = \overline{K} \tag{69}$$

$$\overline{F} = \overline{F}_1 B \tag{70}$$

$$\overline{H} = \overline{H}_1 \overline{K} \tag{71}$$

$$A\overline{F}_1 + \overline{G}q^{-k} = A_m \overline{H}_1 \tag{72}$$

Since the closed-loop transfer function is normally specified such that its steady-state gain is unity, $B_m(1)/A_m(1) = 1$, the constant $\overline{K}$ is given by

$$\overline{K} = A_m(1) \tag{73}$$

Since all of the process zeros are canceled by controller poles, the resulting controller is not suitable for processes with nonminimum-phase or poorly damped zeros.

Case 2. No Process Zeros Are Canceled. For this case, the closed-loop zeros are equal to the process zeros (i.e., $B = B_m$). Thus, in Eq. 62, $B^+ = 1$ and Eqs. 64, 65, 67, and 68 become, respectively,

$$B_m = B_{m1}B = B_{m1}B^- \tag{74}$$

$$\overline{F} = \overline{F}_1 \tag{75}$$

$$\overline{H} = \overline{H}_1 B_{m1} \tag{76}$$

$$A\overline{F} + \overline{G}Bq^{-k} = A_m \overline{H}_1 \tag{77}$$

From Eq. 74, since the zeros of B_m equal the zeros of B, B_{m1} is a constant. So the steady-state gain of the closed-loop transfer function is 1.0, B_{m1} is specified as

$$B_{m1} = \frac{A_m(1)}{B(1)} \tag{78}$$

Here, since none of the process zeros is cancelled, the resulting controller is satisfactory also for processes with nonminimum-phase or poorly damped zeros.

A disadvantage of the pole-zero design procedure is that the controller design calculations, whether given in the general form of Eq. 68 or in the form for one of the special cases above, can be time-consuming. To avoid these controller design calculations, the algorithm can be written as an implicit algorithm by combining Eq. 1, with $\xi = d = 0$ and Eq. 68 to obtain

$$A_m H_1 y(t) = q^{-k} B^- [\overline{F}u(t) + \overline{G}y(t)] \tag{79}$$

The controller design calculations for this alternate process model are trivial, since $\overline{F}$ and $\overline{G}$ appear directly in the model. For implementation of the algorithm in this form, the parameters of the model of Eq. 79 are estimated on-line. However, since Eq. 79 is nonlinear in the parameters, the parameters of $\overline{F}$ and $\overline{G}$ are not easily determined. The estimation problem may be simplified considerably for the case considered previously in which all of the process zeros are cancelled ($B^- = 1$). Using this approach, the self-tuning controller presented by Clarke and Gawthrop (1975) may be obtained (Åström and Wittenmark, 1980). Notice that the case in which none of the process zeros is cancelled ($B^+ = 1$) does not simplify the estimation problem associated with the implicit algorithm.

Dahlin's controller

Dahlin's (1968) controller can also be derived from the general pole-zero controller design procedure presented by Åström and Wittenmark (1980). For Dahlin's controller, the orders of $A(q^{-1})$ and $B(q^{-1})$ are assigned so that

$$m = n - 1 \tag{80}$$

A first-order, closed-loop transfer function is specified for Dahlin's controller; in Eq. 59,

$$A_m(q^{-1}) = 1 - e^{-\Delta t/\tau_c} q^{-1} \tag{81}$$

where Δt is the sampling interval and τ_c is the desired time constant of the closed-loop process, which can be used as a tuning parameter. This controller corresponds to case 1 of Åström and Wittenmark's general procedure in which all process zeros are cancelled. Thus, with A_m defined as above, it is clear from Eqs. 69 and 73 that

$$B_m = 1 - e^{-\Delta t/\tau_c} \tag{82}$$

The controller design equation for this case is given by Eq. 77. H_1 is selected to be A and the controller design equation becomes

$$A\overline{F}_1 + \overline{G}q^{-k} = A_m A \tag{83}$$

Dahlin's controller is then obtained from Eq. 83 by specifying the order of $\overline{F}_1$ to be k and the order of $\overline{G}$ to be n. With these specifications, the number of simultaneous equations represented by Eq. 83 is one less than the number of unknowns. Dahlin's controller (G_{DC}) results by specifying the extra unknown such that $\overline{G} = \overline{H}$. Then, solving the set of simultaneous equations represented by Eq. 83 yields

$$G_{DC} = \frac{\overline{G}(q^{-1})}{\overline{F}(q^{-1})} = \frac{(1 - e^{-\Delta t/\tau_c})A(q^{-1})}{[1 - e^{-\Delta t/\tau_c}q^{-1} - (1 - e^{-\Delta t/\tau_c})q^{-k}]B(q^{-1})} \tag{84}$$

Note that by initially requiring that $\overline{G} = \overline{H}$, the controller design calculations become trivial, since expressions for $\overline{F}_1$ and $\overline{G}$ may be obtained directly from Eq. 72.

Unfortunately, controllers that cancel all of the process zeros are not suitable for nonminimum-phase processes or processes with poorly damped zeros. Thus, Dahlin's controller is unstable for nonminimum-phase processes, and when applied to processes with poorly damped zeros, G_{DC} exhibits the undesirable phenomenon known as ringing. Ringing is the term used to describe excessive oscillation of the controller output, which occurs with some discrete controllers when they are applied to processes with zeros having poor damping. In the presence of ringing, the process output may be at the set point at the sampling instants, but examination of the process output between sampling times reveals that it too oscillates. The conventional approach to reduce ringing is to set $q = 1$ in the ringing term of the controller transfer function (Touchstone and Corripio, 1977). However, Vogel (1982) has shown that this procedure may still give unsatisfactory control, especially for large ratios of dead time to time constant and/or high-order processes. For low-order models, Dahlin's controller can be suitable for use in adaptive control (Touchstone and Corripio, 1977; Hoopes et al., 1983).

Vogel-Edgar controller

Vogel and Edgar (1982a) developed a pole-zero placement controller based on case 2 of Åström and Wittenmark's general pole-zero design procedure. For this controller (G_{VE}) let

$$m = k_2 - 1$$

$$k = k_1 + 1 \tag{85}$$

where k_1 and k_2 are used to denote the expected range for dead time variations, $k_1 \leq k \leq k_1 + k_2$. The Vogel-Edgar controller uses the same $A_m(q^{-1})$ as Dahlin's controller, i.e., Eq. 81. Since $G_{VE}(q^{-1})$ corresponds to the special case where none of the process zeros is cancelled, B_m is determined from Eq. 78 to be

$$B_{m1} = \frac{1 - e^{-\Delta t/\tau_c}}{\sum_{i=0}^{k_2-1} b_i} \tag{86}$$

For this case, the controller design equation is given by Eq. 82 and yields $G_{VE}(q^{-1})$ when the order of $\bar{F}$ is $k_1 + k_2$, the order of $\bar{G}$ is n, and $\bar{H}_1 = A$. With these specifications the controller design equation is

$$A\bar{F} + \bar{G}Bq^{-k} = A_m A \tag{87}$$

The number of simultaneous equations represented by Eq. 93 is one less than the number of unknowns. To obtain $G_{VE}(q^{-1})$, the extra unknown is specified by setting $\bar{G} = \bar{H}$. Solution of the set of simultaneous equations given by Eq. 93 then yields the controller below:

$$G_{VE} = \frac{\bar{G}}{\bar{F}} = \frac{(1 - e^{-\Delta t/\tau_c})(1 + a_1 q^{-1} + \cdots + a_n q^{-n})}{(1 - e^{-\Delta t/\tau_c} q^{-1})\left(\sum_{i=0}^{k_2-1} b_i\right) - (1 - e^{-\Delta t/\tau_c})\,\bar{B}} \tag{88}$$

where

$$\bar{B} = (b_0 + b_1 q^{-1} + \cdots + b_{k_2-1} q^{-k_2+1}) q^{-k_1-1}$$

As with Dahlin's controller, the controller design calculations become trivial by initially requiring that $\bar{G} = \bar{H}$, because $\bar{F}$ and $\bar{G}$ may be solved for directly in Eq. 77.

Controllers that cancel none of the process zeros can be applied to processes that are either nonminimum-phase or that have poorly damped zeros. Therefore, unlike Dahlin's controller, $G_{VE}(q^{-1})$ has no unstable poles for nonminimum-phase processes and is also suitable for processes with poorly damped zeros. For a direct comparison of Dahlin's controller with G_{VE}, see Vogel (1982). Note that both G_{VE} and G_{DC} eliminate offset, since the closed-loop transfer function has a steady-state gain of unity. There are cases when G_{VE} gives sluggish or inadequate performance, requiring modification of the algorithm (McDermott and Mellichamp, 1983). However, the controller calculation in Eq. 88 is analytical in form and does not require solution of simultaneous equations, in contrast to other pole placement controllers.

Applications of pole placement controllers

The number of applications of pole placement adaptive controllers relative to those of the self-tuning controllers discussed earlier is small. Very little large-scale experimental or commercial testing has been reported at this time, with most experimental work carried on a laboratory or pilot scale. Commercial applications have been discussed by Corripio and Tompkins (1981), Proudfoot (1983) and Hoopes et al. (1983). Table 3 lists applications classified relative to type, denoting the nature of the test (simulated or experimental).

Multivariable self-tuning controller design based on pole placement

Prager and Wellstead (1980) developed a self-tuning regulator for multivariable systems based on pole placement. The multivariable controller development and application procedure are analogous to that for the SISO pole placement algorithm. The multivariable process model is given by Eq. 21. The multivariable pole placement controller has the form [for $y_r(t) = 0$]

$$u(t) = \bar{G}\bar{F}^{-1}y(t) \tag{89}$$

where the controller polynomial matrices $\bar{G}$ and $\bar{F}$ are defined as

$$\bar{F}(q^{-1}) = I + \bar{F}_1 q^{-1} + \cdots + \bar{F}_n q^{-n}$$

$$\bar{G}(q^{-1}) = \bar{G}_0 + \bar{G}_1 q^{-1} + \cdots + \bar{G}_m q^{-m}$$

With the controller given by Eq. 89, the closed-loop relationship is

$$y(t) = \bar{F}[A\bar{F} - q^{-k}B\bar{G}]^{-1} C\xi(t) \tag{90}$$

Table 3. Applications of Pole Placement Adaptive Controllers

Absorption/Desorption	Furnace
Ristić et al. (1983) (e)	Corripio and Tompkins (1981) (e)
Boiler	Heat Exchanger and Heating Systems
Hoopes et al. (1983) (e)	Vogel (1982) (e)
Chemical Reactor	Moghtader and Warwick (1983) (e)
Koutchockali et al. (1984 (e)	Moghtader et al. (1984) (e)
Kwalik and Schork (1985) (s)	Liquid Level
McDermott and Mellichamp (1983, 1984a, 1984b) (s) and (e)	Prager and Wellstead (1980) (e)
Touchstone and Corripio (1977) (s)	Wellstead et al. (1979) (e)
Distillation Column	pH Control
Gerry et al. (1983) (e)	Proudfoot (1983) (e)
Vogel (1982) (s)	

(s) simulated
(e) experimental

The controller matrices $\bar{F}$ and $\bar{G}$ are selected so that the closed-loop poles correspond to those specified by the polynomial matrix T, where

$$T(q^{-1}) = I + T_1 q^{-1} + \cdots + T_n q^{-n} \quad (91)$$

The closed-loop poles are selected to give appropriate damping characteristics but are usually application-specific. Thus, F and G are determined by solving the following set of simultaneous equations.

$$A\bar{F} - q^{-k}B\bar{G} = CT \quad (92)$$

With $\bar{F}$ and $\bar{G}$ determined as above, the closed-loop transfer function is

$$y(t) = \bar{F}T^{-1}\xi(t) \quad (93)$$

The control law, Eq. 89, is implemented as follows:

$$\bar{F}^* u(t) = \bar{G}^* y(t) \quad (94)$$

Where $\bar{F}^*$ and $\bar{G}^*$ have the same form as $\bar{F}$ and $\bar{G}$, respectively, and are determined from

$$\bar{F}^*\bar{G} = \bar{G}^*\bar{F} \quad (95)$$

Equation 95 also yields a set of simultaneous equations.

The features of the multivariable pole placement controller are similar to those of the SISO pole placement controller. The multivariable controller is not sensitive to nonminimum-phase processes and it may be detuned to avoid excessive control action. The controller also provides dead time compensation and allows for different time delays between the input-output combinations and allows for variable time delays. Both of these desirable features are obtained by increasing the order of the polynomial matrix $B(q^{-1})$. Note that this similarly increases the order for all of the polynomials represented by $B(q^{-1})$, so it is analogous to the approach used with the SISO pole placement controller (see the preceding section on the Vogel-Edgar controller).

Prager and Wellstead (1980) have suggested that it is good practice to incorporate integrators in each control loop. While this ensures zero offset, the controller is not particularly well-suited for servo control, because this design procedure does not necessarily lead to a controller that decouples the interactions between the input-output pairs. Further, frequent set-point changes tend to detune the controller.

Implementation of the multivariable pole placement controller as a self-tuning controller requires the following steps each iteration:

1. Estimation of the model parameters in Eq. 89.
2. Calculation of the controller parameters from Eqs. 92 and 95.
3. Calculation of the control action from Eq. 94.

The procedure is often simplified by setting $C(q^{-1}) = I$. However, this explicit algorithm has the significant disadvantage that the controller design calculations are likely to require a considerable amount of computation time, since they involve the solution of two sets of simultaneous equations each iteration. Since the order of the expanded B polynomial depends on the process dead time, the algorithm is especially not attractive for processes with large dead times.

One of the disadvantages of the design procedure of Wellstead et al.'s technique is the estimation of many unnecessary model parameters, since the order of the $B(q^{-1})$ polynomials must be selected large enough to include the ranges of all of the process delays. In contrast, Vogel and Edgar (1982b) developed an algorithm in which the number of parameters in each element of $B(q^{-1})$ depends only on the expected range for the dead time of that element. Thus their model has the advantage that it is likely to require estimation of fewer parameters compared to the model used by Wellstead et al. (1979).

For application with the Vogel-Edgar multivariable dead time compensator, G_{VE}, the process model is as follows:

$$y(t) = G(q^{-1})U(t) \quad (96)$$

where

$$G(q^{-1}) = A^{-l}(q^{-1})B(q^{-1}) \quad (97)$$

and $A(q^{-1})$ and $B(q^{-1})$ are defined in Eq. 20.

In analogy with the approach used with multivariable dead-time compensators (Ogunnaike and Ray, 1979), a process model without delays, $G_1(q^{-1})$, is obtained from $G(q^{-1})$ by replacing each element of $B(q^{-1})$ by the corresponding sum of the coefficients in each element of $B(q^{-1})$. Thus the process model without delays is

$$y(t) = G_1(q^{-1})u(t) \quad (98)$$

where $G_1(q^{-1})$ is given by

$$G_1(q^{-1}) = A^{-1}(q^{-1})\Sigma B(q^{-1}) \quad (99)$$

$A(q^{-1})$ is defined as in Eq. 22 and the elements of $\Sigma B(q^{-1})$ are defined as

$$\sum B_{ij}(q^{-1}) = \left(\sum_{\ell=1}^{r_{ij}} b_\ell^{ij}\right) q^{-1} \quad (100)$$

The closed-loop transfer function for the MIMO system is

$$y(t) = GG_1^{-1}T_1 y_r(t) \quad (101)$$

where $T_1(q^{-1})$ is a diagonal matrix selected to reduce interactions between the input-output pairs. The components of T_1 are

$$T_{1ii} = \frac{(1 - e^{-\Delta t/\tau_{ci}})q^{-1}}{1 - e^{-\Delta t/\tau_{ci}}q^{-1}} \quad (102)$$

where τ_{ci} is a tuning parameter that determines the response time of each loop. The solution for G_{VE} to achieve the desired response in Eq. 101 is

$$G_{VE} = [G_1 - \bar{F}G]^{-1}\bar{F} \quad (103)$$

This algorithm does not require the simultaneous solution of many equations to synthesize the controller and only needs to invert an $N \times N$ matrix (Eq. 103) at each time step, where N is the dimension of y and u.

The above multivariable adaptive controller/dead-time compensator is analogous to the SISO algorithm G_{VE} discussed earlier, and has similar properties: explicit algorithm with direct controller updating (no controller design calculations), on-line tuning parameter for each input-output pair, stable with nonminimum-phase processes (requiring no detuning), and servo design based on pole-zero placement with integral action. Additionally, the multivariable algorithm allows for different time delays between the input-output combinations. This feature does not increase the complexity of the algorithm or the number of process model parameters that must be estimated. Simulation results by Vogel (1982) and Kwalik and Schork (1985) have shown that the MIMO controller does perform satisfactorily for most operating conditions.

Recently McDermott (1984), and McDermott et al. (1984c) have reported a pole placement algorithm that incorporates the best features from the Wellstead-Prager and Vogel-Edgar approaches and successfully treats unstable as well as nonminimum processes. The main improvement arises from on-line optimization of the closed-loop poles to adjust the closed-loop response and account for inexact decoupling of the true process. As in Vogel and Edgar's approach, static decoupling is achieved in this algorithm, and a least squares approximation to dynamic decoupling is realized. McDermott and Mellichamp (1984b) reported a successful experimental application of this MIMO self-tuner for a packed-bed reactor.

The connection between the MIMO minimum variance and pole placement algorithms has recently been explored by Dugard et al. (1984) and Elliott and Wolovich (1984). Using the interactor matrix approach for describing the MIMO system, Elliott and Wolovich have shown that an indirect adaptive control strategy can be used with plant data, while some difficulties arise with the direct methods. They also have explored ways to reduce the size of the parameter estimation problem for MIMO systems.

Stable Adaptive Control

In concept, the design of an adaptive system is simple. A very natural approach is to combine a particular parameter estimation technique (e.g., recursive least squares, projection type estimation algorithms—see the parameter estimation law, below) with a control law (dead-beat, pole-placement algorithm, etc.). In this manner one can generate a large number of algorithms, depending upon the specific combination of estimation and control laws used. In the present section the focus of attention is those combinations of estimation and control laws that are motivated by a stability requirement and therefore have proven convergence and stability properties. Specifically, this section explains the main stability results of a particular combination of parameter estimation and control law.

Within the past several years there have been a number of important results contributing to the previously unresolved issue of global stability of adaptive control systems. The key question concerning stability of adaptive systems is how to ensure that the regressor or the input-output vector, $\psi(\cdot)$, consisting of present and past values of plant inputs and outputs, remains bounded or finite at all times. This problem proved elusive until the late 1970's. One of the first important results on stability of adaptive systems was due to Monopoli (1974). His scheme claimed to show stability of an augmented error-based algorithm by using Lyapunov's stability criterion. The idea of an augmented signal as introduced by Monopoli to avoid differentiation of certain signals in the adaptive control algorithm, has proved to be important and is an essential element of any adaptive system for which global stability can be assured. At the present time, it is still not known if Monopoli's algorithm guarantees stability except for the special case where only one process model parameter is to be estimated (Feuer and Morse, 1978).

An important step forward was made when Goodwin et al. (1978a) showed that a relatively simple adaptive algorithm applicable to MIMO discrete-time systems would provide global stability. Egardt (1979a, b) also proposed an adaptive algorithm with a rather complicated and technically involved proof of stability. The work of many other authors has also contributed significantly to the overall question of stability of adaptive systems: Morse (1980), Fuchs (1980), Narendra and Lin (1980), Martin-Sanchez et al. (1981), Peterson and Narendra (1982), Elliott (1982), Elliott and Wolovich (1982), Martin-Sanchez (1984), Kreisselmeier and Narendra (1982), and Samson (1983).

In the different adaptive schemes proposed by these authors, the choice of the parameter estimation law is the key step and is mainly motivated by stability and convergence analysis. The results reported by these authors show that the regressor vector, $\psi(\cdot)$, is bounded (thus proving stability) and simultaneously establish convergence properties for the parameters and asymptotic convergence (in the deterministic case) of the control error to zero.

There are several reasons for looking at the problem of stability and convergence of adaptive algorithms. First, a convergence and stability result helps distinguish between good and bad algorithms. Second, such a result identifies the important ingredients of a stable algorithm and thus helps to suggest ways in which an algorithm might be improved. Last, although the proof of stability of such algorithms is only valid under ideal conditions, such a result does give some credibility to the algorithm. An excellent overview of the current status of convergence theory for adaptive systems has been given by Goodwin et al. (1984) as well as other recent articles in the *Automatica* (Sept. 1984) special issue on adaptive control.

Model reference adaptive control

Historically, the first attempts in the design of stable adaptive systems were in the area of model reference adaptive control (MRAC) systems design. In MRAC the basic objective is to make the output of an unknown plant asymptotically approach that of a given reference model. The model reference idea was originally proposed by Whitaker et al. (1958) and was further developed by Parks (1966), Monopoli (1974), and Landau (1974). The book by Landau (1979) provides a comprehensive account of work in this area up to 1977. More recent developments in this area have been reviewed by Narendra and Peterson (1980) and in various papers available in the proceedings of the 1979, 1981, 1983, and 1985 Yale workshops on adaptive systems.

During the 1970's much of the earlier focus on stable adaptive systems was based on the MRAC system formulation (Landau, 1974; Monopoli, 1974), using the stability methods of Lyapunov or the hyperstability criterion of Popov. Many of these results showed asymptotic convergence of the plant output to the reference model output. However, the key stability question of boundedness of the input and output vector was not addressed properly (Johnstone and Anderson, 1982) and remained unresolved until relatively recently.

Many of the earlier MRAC results were based on the state-space formulation and required knowledge of the state-space model and measurements of all the state variables (Parks, 1966). Subsequent methods still based on the state-space approach were concerned with unknown processes and their identification, state estimation via adaptive observers, and control (Landau, 1979). However, from a practical point of view the adaptive control problem based on input-output signals is intuitively more appealing and has proved to be the main focus of interest in the design of stable MRAC controllers (Narendra and Valvani, 1979).

One possible control configuration under the MRAC philosophy is shown in Figure 5. Note that the particular control configuration in this figure corresponds to a direct MRAC system. The input and output of an unknown, linear, time-invariant plant are $u(\cdot)$ and $y(\cdot)$, respectively. A reference model representing the desired behavior and a reference or set-point trajectory $y_r(\cdot)$ are specified, which result in a model output $y_m(\cdot)$. From all the available data it is desired to adjust the feedback and feedforward control parameters such that the error $[e(t) = y(t) - y_m(t)]$, tends to zero asymptotically. The key technical problem is to determine the structure of the adjustment mechanism such that the overall system is globally stable, i.e., the plant input, $u(t)$, and output, $y(t)$, remain bounded for all t and the error, $e(t)$, goes to zero as $t \rightarrow \infty$. This problem of showing global stability is a nontrivial one. It remained an open question for many years but was finally resolved in the late 1970's by independent work on this and related problems by Egardt (1979b), Goodwin et al (1978a, b), Narendra and Valavani (1978), Fuchs (1980), and Narendra and Lin (1980).

The results of Morse (1980) and Goodwin et al. (1978a) among others, although based on non-MRAC adaptive controllers, are important in the context of MRAC stability since the MRAC control problem fits naturally within the general framework of their stability analysis—i.e., the MRAC problem can easily be cast into a form suitable for stability analysis using their results. It has also been shown by several authors (Ljung and Landau (1978), Egardt (1979a, b). Åström (1980a), Johnson (1979), Shah and Fisher (1980)), that many of the MRAC and STR type schemes are similar either by being special cases of one algorithm or by giving rise to identical error equations.

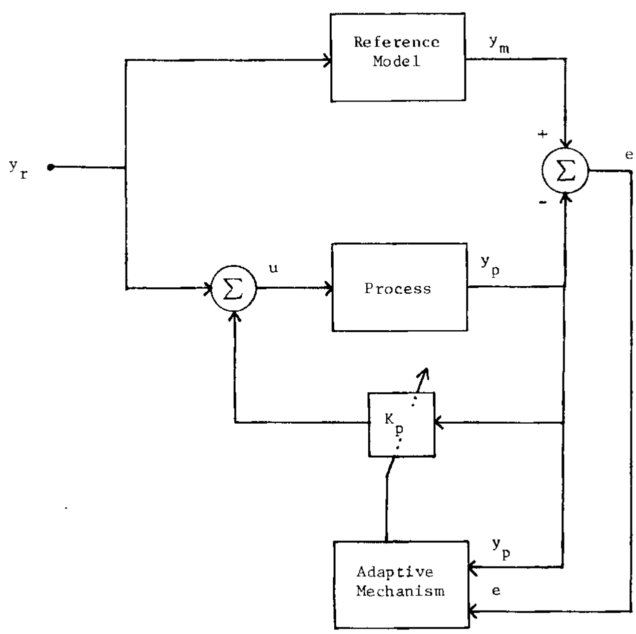

Figure 5. Typical configuration of a direct MRAC system.

Model reference adaptive controller design

The plant to be controlled is assumed to be representable by the following deterministic ARMA (auto-regressive, moving average) model:

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) \tag{104}$$

where $A(\cdot)$ and $B(\cdot)$ are polynomials of order n and m, respectively, in the backward shift operator as defined in Eq. 2. A reference model represents the behavior desired from the plant when it is augmented with a suitable controller. The reference model input, $y_r(t)$, is tracked by the output, $Y_m(t)$, in the following manner:

$$E(q^{-1})y_m(t) = q^{-k}H(q^{-1})y_r(t) \tag{105}$$

where $E(\cdot)$ and $H(\cdot)$ are user-specified polynomials of order p. Defining the relationship

$$E(q^{-1}) = F(q^{-1})A(q^{-1}) + q^{-k}G(q^{-1}) \tag{106}$$

where $F(\cdot)$ and $G(\cdot)$ are unique polynomials of order $k + 1$ and $n + 1$, respectively, and combining it with Eq. 104 allows $y(\cdot)$ to be expressed in a predictive form as:

$$E(q^{-1})y(t) = q^{-k}G(q^{-1})y(t) + q^{-k}F(q^{-1})B(q^{-1})u(t) \tag{107}$$

If $y(\cdot)$ is to be equal to $y_m(\cdot)$, then the lefthand sides of Eqs. 105 and 107 must be equal. This gives the control law:

$$G(q^{-1})y(t) + F(q^{-1})B(q^{-1})u(t) = H(q^{-1})y_r(t) \tag{108}$$

This control law will ensure that the process output will track the reference model output. To make this control law adaptive, one simply needs to replace the true values of the parameters in the control law by their estimated values. The predictor, Eq. 107, can be used to estimate coefficients of polynomials G and FB, and these estimated parameters can then be used to calculate $u(t)$ in Eq. 108. A number of different algorithms can be used to estimate the parameters in Eq. 107. However, one of the simplest algorithms is the following gradient scheme whose convergence properties have been studied using a Lyapunov-type analysis (Goodwin et al., 1980).

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\psi(t-1)}{C + \psi(t-1)^T\psi(t-1)}[y_a(t) - \psi(t-1)^T\hat{\theta}(t-1)] \tag{109}$$

where

$$\psi(t-1)^T = [y(t-k), \ldots y(t-k-n), \\ u(t-k), \ldots u(t-k-m)]$$

$$y_a(t) = E(z^{-1})y(t) \quad \text{and} \quad C > 0.$$

The complete proof of convergence is omitted here for the sake of brevity. For details regarding the proof the reader is referred to Goodwin and Sin (1984). It should be noted here that the MRAC controller can also be interpreted as a special case of the adaptive pole placement controller if $A_m(z^{-1})$ in Eqs. 66 and 73 is chosen as $E(z^{-1})$.

Applications of MRAC systems

Several applications of model reference adaptive control systems have been reported in the literature. Most of these have been summarized in the extensive literature review on applications of adaptive control by Parks et al. (1980). A majority of these applications are in the electromechanical area, for example in control of a DC-drive system, and for tracking an optical telescope. Two notable applications of MRAC techniques in chemical process control area have been on a pilot-scale double-effect evaporator (Oliver et al, 1974) and on a packed-bed tubular reactor (Tremblay and Wright, 1977). Several newer applications of model reference adaptive control have also been reported in the recent proceedings of the IFAC workshop on Adaptive Control (1983) and the Proceedings of the Yale workshop on applications of Adaptive Systems Theory (1979, 1981, 1983, 1985). In general, the number of reported significant applications of MRAC techniques is quite small, in fact, hardly any at all in the process industries. Table 4 summarizes these and other applications of model reference adaptive control.

A stable adaptive controller

The main focus of interest in this section is an overview of the recent results on stability and convergence analysis of adaptive systems. Our reference point will be the results of Goodwin et al. (1980) for the deterministic case and Martin-Sanchez et al. (1981) for the more general stochastic case, which includes external noise and unmeasured disturbances. For simplicity and clarity in explanation, the algorithm and stability analysis will be illustrated by application to a SISO system.

The design of a stable adaptive control law is based on the assumption that the physical process to be controlled can be described by the following stable-inverse or minimum-phase, discrete, ARMA model:

$$y(t) = \theta_0^T \psi(t-1) \tag{110}$$

where θ_0 is the process parameter vector, i.e., $\theta_0^T = [a_1, a_2, \ldots, a_n, b_1, b_2, \ldots]$; and $\psi(\cdot)$ is the process input-output vector, Eq. 3. Now we define the one-step-ahead control or tracking error as

$$\begin{aligned} e(t+1) &= y_r(t+1) - y(t+1) \\ &= y_r(t+1) - \theta_0^T \psi(t) \end{aligned} \tag{111}$$

By choosing $u(t)$ [one of the elements of $\psi(t)$] to satisfy the following equation

$$\phi_0^T \psi(t) = y_r(t+1) \tag{112}$$

it is clear that the control or tracking error will be identically zero. However, since θ_0 is unknown we replace it by an estimate $\hat{\theta}(t)$, i.e.

$$\hat{\theta}(t)^T \psi(t) = y_r(t+1) \tag{113}$$

$\hat{\theta}(t)$ can be obtained from a recursive parameter estimation law.

Parameter estimation law

The following parameter estimation law is proposed for the adaptive algorithm. This is a slightly modified form of the gradient algorithm, Eq. 109. Its choice has been motivated by the stability and convergence analysis:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\alpha(t)\psi(t-1)}{1 + \alpha(t)\psi(t-1)^T\psi(t-1)} \cdot [y(t) - \hat{\theta}(t-1)^T\psi(t-1)] \tag{114}$$

where $\alpha(t)$ is a finite, nonnegative, user-selected parameter that determines the speed of convergence.

The parameter estimation law plays a crucial role in determining the stability of the overall system and is one of the key components of an adaptive system. This parameter update law is also a particular case of the general form of the recursive estimation law expressed in words early in this paper.

The structural form of the above estimation law is simple, and the choice of the algorithm gain, $\alpha(t)/1 + \alpha(t)\psi(t-1)^T\psi(t-1)$, to correspond to one of the bracketed terms of the general recursive estimation law in a stability-based adaptive algorithm is usually motivated by the requirement of global stability of the overall system. The particular form of estimation law, Eq. 114, used in the illustration here is known as a projection algorithm and can be interpreted geometrically as follows: The new estimate $\hat{\theta}(t-1)$ is an orthogonal projection of $\hat{\theta}(t-1)$ onto the hypersurface $y(t) - \theta^T\psi(t-1) = 0$. Note that the choice of

Table 4. Applications of Stable Adaptive Control Methods MRAC Control Methods

Chemical Reactors	Furnace
Tremblay and Wright (1977)	Landau and Muller (1976)
Koutchoukali et al. (1983)	Dahhou et al. (1983)
Kiparissides and Shah (1983)	Heat Exchangers
Dochain and Bastin (1984)	Lozano and Bonilla (1983)
Cluett et al. (1982)	Liquid Levels
Distillation Columns	Wan et al. (1979)
Wiemer et al. (1983)	pH Control
Martin-Sanchez and Shah (1984)	Rodellar and Martin-Sanchez (1980)
Evaporator	Utility Systems (Power Stations)
Oliver et al. (1974)	Irving and Dang Van Mien (1979)
Elicabe and Meira (1983)	Irving et al. (1979)
Song et al. (1983a,b)	Kalnitsky and Mabius (1979)

objective error in this algorithm implies that for the algorithm to converge it is not necessary for $\hat{\theta}(t)$ to converge to θ_0; i.e., $\psi(t-1)$ can be orthogonal to $[\theta_0 - \hat{\theta}(t-1)]$.

Many other algorithms, e.g., the orthogonalized projection algorithm or the least squares algorithm, can be cast into the above basic structure and hence be used in a stability-based adaptive algorithm (Goodwin and Sin, 1984). Each algorithm has different convergence properties and the choice depends on the particular environment in which it is used. In fact the reader should be aware of the importance of problem-specific modifications to the basic algorithm to improve its performance.

Stability

The key requirement in the proof of global stability of the deterministic adaptive system is to show that the process input and output valves of an unknown plant with a particular estimation and control law, remain bounded for all values of t, and furthermore that the tracking or control error $e(t) \rightarrow 0$ as $t \rightarrow \infty$. The following simplified statement of the stability theorem summarizes the key stability issues applicable to adaptive systems.

Theorem

Subject to the following assumptions:

a. An upper bound on the order of ARMA representation of the process, i.e., n and m are known.

b. The process delay, k, is known.

c. The unknown process is minimum-phase, i.e., it has a stable inverse.

Then the following properties are true if the estimation law, Eq. 114, is combined with the control law, Eq. 113, and applied to an unknown process, Eq. 110:

(i) $\|\psi(t)\| < \infty \ \forall \ t$, i.e., for a bounded reference input, $y_r(\cdot)$, the input-output or the regressor vector is always bounded. (Stability is assured.)

(ii) $\lim_{t\rightarrow\infty} e(t) = \lim_{t\rightarrow\infty} [y(t) - y_r(t)] = 0$
(asymptotic tracking property)

The first property is obtained from the assumption of a stable-invertible or minimum-phase system and is a key technical lemma due to Goodwin et al. (1980). One interesting result in showing property (i) is the following convergence result:

$$\|\hat{\theta}(t) - \theta_0\| \leq \|\hat{\theta}(t-1) - \theta_0\|$$

This result shows that the Euclidean norm of the parameter estimation error vector is a bounded nonincreasing function. Note that it is not proved or claimed that $\hat{\theta}(t)$ converges to θ_0. The system properties assumed here with the estimation and control laws are enough to ensure global stability and asymptotic tracking, and true parameter convergence is not required. (In order to establish parameter convergence, if this is at all required—e.g., in adaptive filtering algorithms—one needs to impose the condition of persistent excitation on the control input signal).

To date, the stability of various classes of adaptive algorithms based on one-step-ahead or predictive control and model reference adaptive control, as described above, has been established. However, one of the unresolved problems on the issue of the stability of adaptive systems is the design of a globally stable controller for pole-positioning. The difficulty in showing global stability of pole-positioning is usually one of checking if the estimated plant numerator and denominator polynomials have any comon factors, i.e., if $A(\bar{z}^1)$ and $B(\bar{z}^1)$ are relatively prime, and if these are true parameters, then coping with the problem of an uncontrollable and/or nonstabilizable plant. This difficulty can be resolved if it can be shown that the estimated plant parameters can converge to their true values. True parameters convergence has been established by applying an external set-point perturbation or a persistently exciting set-point signal to the plant (Anderson and Johnson, 1982). The remaining difficulty in the proof of global stability of adaptive pole-positioning algorithms is to then show boundedness of the system variables independent of parameter convergence.

Stable adaptive control in the presence of noise and disturbances

The above stability result for deterministic systems is of great significance, but from an application point of view it is lacking in realistic assumptions, such as the existence of noise and unmeasured disturbances acting on the system. The overall stability of a minimum-phase process in the presence of bounded unmeasured disturbances and noise has been shown by Martin-Sanchez et al. (1981). No a priori information other than boundedness of these (disturbance and noise) signals is assumed and therefore their result is applicable to stochastic systems. This result is based on a novel discrete-time representation of the process and leads to a stability analysis using the same projection estimation algorithm as in Eq. 114, but with special criteria for setting values of the adaptation gain $\alpha(t)$.

Shorter and more direct proofs of analogous stability results have been obtained both for continuous (Peterson and Narendra, 1982) and discrete (Samson, 1983) systems, but these proofs also suffer from a condition on the a priori estimate of a process parameter. All of these latter results (Egardt, 1979a, Peterson and Narendra, 1982; Samson, 1983) have considered criteria for continuing or stopping parameter adaptation. They also use as a main argument in their proofs the boundedness of the rate of change of the system input-output signals.

A process with external noise and disturbances can be represented by

$$y(t) = \theta_0^T \psi(t-1) + \Delta(t) \tag{115}$$

where $\Delta(\cdot)$ is the "perturbation," to account for the effect of such variables as bounded unmeasured disturbances plus process and measurement noise on the system output. Since the only assumption on $\Delta(\cdot)$ is that it be bounded, the process model represented by Eq. 115 is very general and can be applied to a broad class of industrial processes. For this case the control law is identical to Eq. 113. However, the provision for incremental control is available should it be necessary to remove the offset in the control error in the presence of constant or sustained disturbances. The estimation law is identical to Eq. 114, but the adaptation gain, $\alpha(t)$, is chosen (in the simplest case) according to the following criterion:

$$\alpha(t) = \begin{cases} 1 \text{ if } |y(t) - \hat{\theta}(t-1)^T \psi(t-1)| > 2\Delta_m \\ \quad \text{(adaptation on)} \\ \text{where } \Delta_m \text{ is the user specified} \\ \quad \text{upper bound on } \Delta \\ 0 \text{ otherwise} \qquad \text{(adaptation off)} \end{cases} \tag{116}$$

Such a criterion to stop parameter adaptation when the prediction error is small is a necessary part of the stability proof (Martin-Sanchez et al., 1981) and appears to be reasonable and intuitive. Note that for the algorithm to converge it is not necessary for $\theta(t)$ to converge to θ_0. In fact with the dead zone feature in the estimation law (Eqs. 114 and 116), only as much convergence of $\theta(0)$ to $\theta(t)$ is required as is necessary to cause the |prediction error| $< 2\Delta_m$. Once this occurs adaptation stops and is restarted only if this latter condition is violated.

Implementation of the adaptive algorithm

To implement the specific algorithms discussed here the following computations have to be carried in the order indicated:

Step 1. Measure current process output, $y(t)$, and formulate $(t-1)$. Compute control error $e(t)$ and the a priori prediction error

$$e(t|t-1) = y(t) - \hat{y}(t|t-1) = y(t) - \hat{\theta}(t-1)^T\psi(t-1).$$

Step 2. Compute the new parameter estimation vector $\hat{\theta}(t)$ using Eq. 114 with or without Eq. 116.

Step 3. Knowing the desired set point $y_r(t+1)$ at the next step, calculate the current control input signal, $u(t)$ from Eq. 113. Repeat Steps 1 to 3 at each sampling instance.

The performance of such an algorithm (Eq. 114 with Eq. 116) is illustrated by considering the experimental application of this algorithm to the composition control of a binary distillation column. Figure 6 shows the performance of this algorithm in comparison to that of a well-tuned PI controller. Additional details and results of other SISO and MIMO adaptive predictive control runs on the same distillation column are given in a recent paper by Martin-Sanchez and Shah (1984).

Applications

Most applications of such stability-based adaptive algorithms have been at universities; only a few have been actual experimental evaluations, usually involving pilot-plant units. An early application by Martin-Sanchez (1977, 1984) involved experimental evaluation of a multivariable adaptive predictive control strategy on a pilot-scale distillation column. Other applications of the same strategy included control of pH in a simulated process (Rodellar and Martin-Sanchez, 1980), and batch polymer reactor applications (Kiparissides and Shah, 1983; Cluett et al., 1982). Illustrative laboratory applications of many of these algorithms have been reported in the textbook by Goodwin and Sin (1984). Goodwin et al. (1982b) have also applied such algorithms on a simulation basis for wastewater treatment and pH neutralization. A summary of these as well as more recent applications of such stable adaptive controllers is outlined in Table 4.

Some unresolved problems

The question of overall stability and convergence of a stochastic self-tuning regulator was resolved by Goodwin et al. (1981). The analysis of stochastic time-varying systems remains an open problem.

Much of the current theory of adaptive systems relies on the assumption that the plant or process to be controlled can be represented by some member of a family of linear, finite-dimensional parametric models. Since few plants are truly of finite order and linear, modeling errors due to unmodeled plant dynamics and nonlinearities are invariably present. Currently, efforts are underway to show and improve the robustness of adaptive systems when operating under such nonideal conditions (Kosut and Johnson, 1984; Ioannou and Kokotovic, 1984).

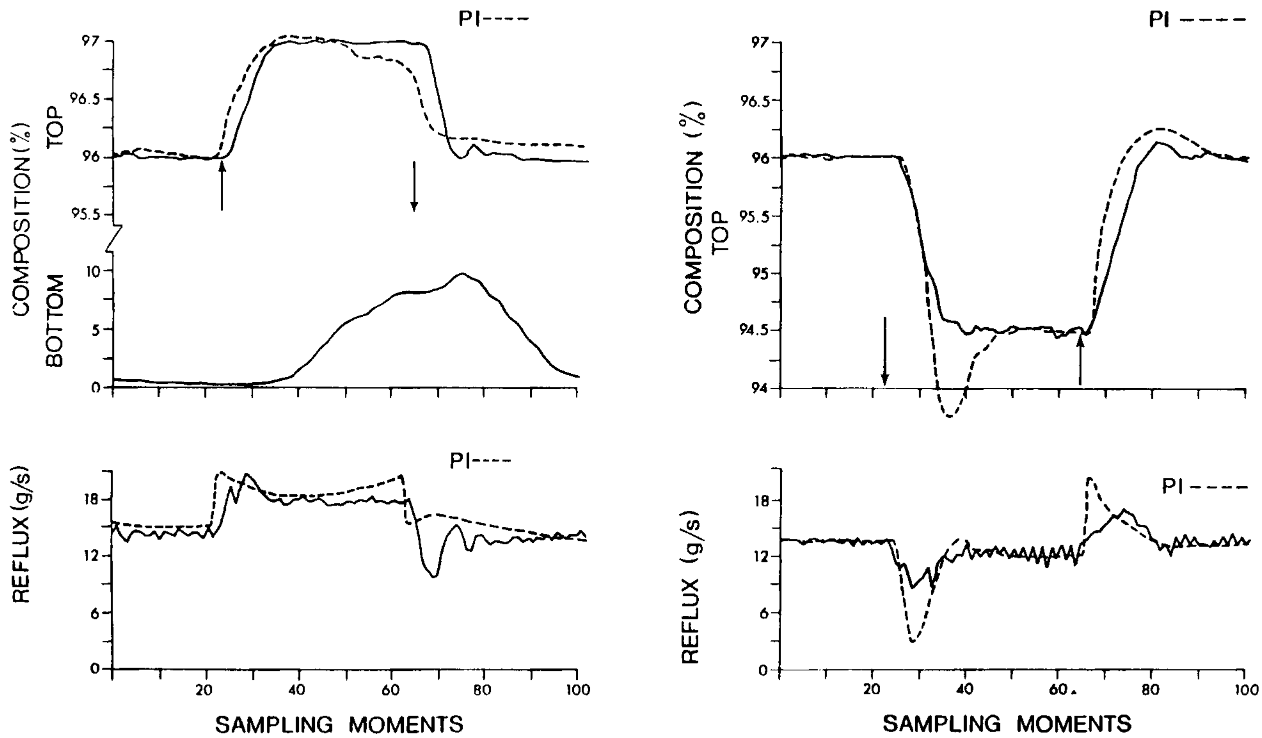

Figure 6. SISO adaptive predictive control of the top composition of a binary distillation column.

Miscellaneous Forms of Adaptive Control

Apart from the most popular forms of adaptive control methods presented in the earlier sections, there are several other control design techniques that may be classified as adaptive systems according to the definition of adaptive control stated in the introduction. Some of these are discussed briefly in the following two sections.

Adaptive control via pattern recognition

Pattern recognition can be used as an alternative to identification in adaptive control. Bristol (1977) has introduced a pattern recognition scheme that is designed to monitor errors between a controlled plant and its reduced order model and to adaptively "infer" or attribute the cause of these residual errors to, e.g., unmeasured disturbances entering the plant. The identification of particular types of transient terms in the plant output or residual errors through specified logic filters, i.e., by pattern recognition, is then used to take the appropriate corrective action. However, the design of a pattern recognition scheme appears to be problem-specific and would require considerable a priori information on the process characteristics.

In 1984 Foxboro announced the availability of a self-tuning PID controller that is based on a so-called expert system approach for adjustment of controller parameters; i.e., the controller in its adaptive state uses many knowledge-based rules built upon tuning observations and tables (Kraus and Myron, 1984). The on-line tuning of K_c, τ_I, and τ_D is based on the closed-loop transient response to a step change in set point. By evaluating the salient characteristics of the response (e.g., the decay ratio, overshoot, and closed-loop period), the controller parameters can be updated without actually finding a new process model. The details of the algorithm, however, are proprietary.

Extremum adaptive control systems

For a class of (usually nonlinear) systems the relationship between the input and output has an extremum, a minimum or maximum point—i.e., for a particular value of input the output may have a maximum or minimum value, and it may be desired to keep the output of a system operating at this extremum value. For example, in the control of the air/fuel ratio for optimal combustion there is an optimal setting for air flow (input) depending on the fuel quality that results in the optimum combustion (output). Such systems can be controlled by an adaptive extremum strategy. The objective in adaptive extremum control systems is to identify the extremum function and then adjust the control input so that the output is at its extremum value. Such a class of systems did attract attention in the 1960's (Blackman, 1962; Jacobs, 1969). In fact one of the first reported adaptive schemes—the "MIT rule" of Whitaker et al. (1958)—can be thought of as an extremal adaptive scheme in which the objective is to measure a performance function and minimize or maximize this objective function in the presence of changing process gains. However, interest in such strategies diminished because of lack of appropriate hardware for implementing them. With the availability of microprocessors, such adaptive strategies are being revived (Sternby, 1980).

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